Supplement: Temporal and spatial Taylor's law: Application to Japanese subnational mortality rates

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This section presents additional plots and numerical results related to the proposed extensions of the classic Taylor's law. Figure A1 shows fitted long-run spatial TL regressions for total, female and male series in each year between 1975–2018, with 95% prediction intervals represented by shaded areas. The ages between 0–100 are highlighted by colors in the rainbow order, i.e., red to purple corresponding to 101 ages in ascending order. For all years between 1975–2018, spatial variances (in $\log_{10}$ scale) are observed to increase with spatial mean values (in $\log_{10}$ scale) but not at a constant rate. The fitted cubic spatial TL regressions capture observations' variation well, and 95% prediction intervals cover nearly all data points.

Figure A2 shows temporal TL fitted models with 95% PIs for Japanese age-specific mortality rates by prefecture from north (Hokkaido) to south (Okinawa).

Within-cluster sum of squares for K-means clustering applied to OLS estimates of the long-run temporal TL slope are presented in Figure A3. The scree plots obtained indicate that a total of 3 or 4 clusters would be appropriate for the female, male, and total series in Japan. To obtain a simple yet informative demonstration, $K = 2$ is selected to generate clustering results shown in Table 2 in the main text.

Figure A4 displays AICc and predicted R^2 values for the total, female and male mortality series as a function of θ, where $\theta \in [0, 1]$ is a weight balancing the spatial and temporal components of $C^{\text{LR spatial}}(u)$ in equation (10) of the main manuscript.

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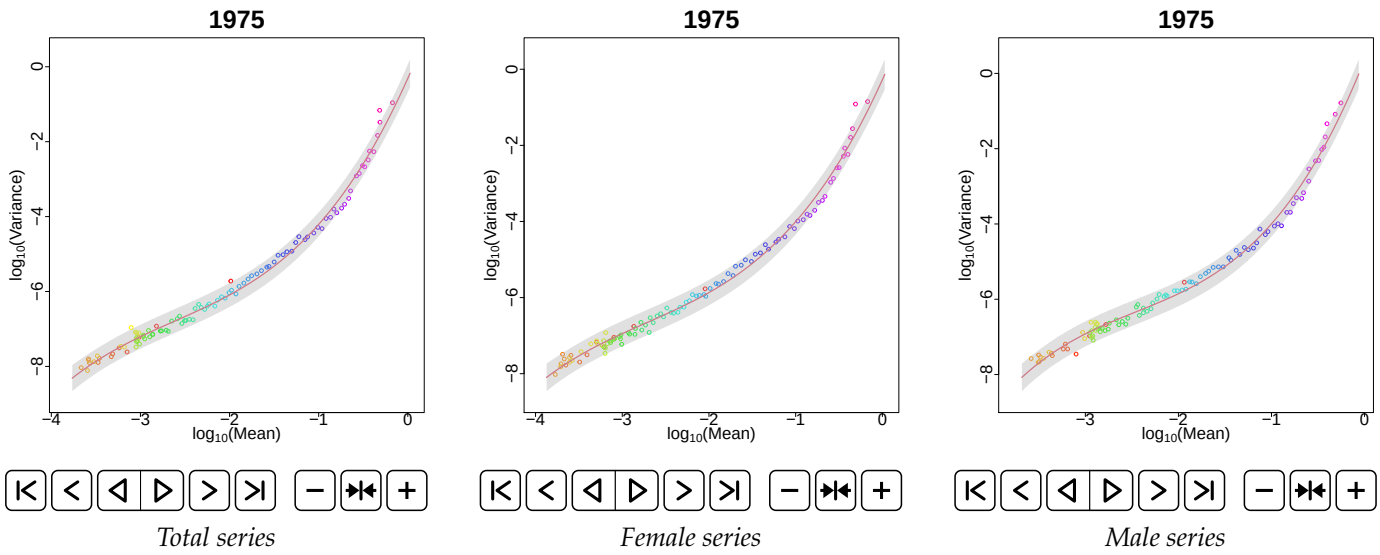

Figure A1: *Spatial TL fitted models with 95% prediction intervals for Japanese age-specific mortality rates by year 1975–2018.*

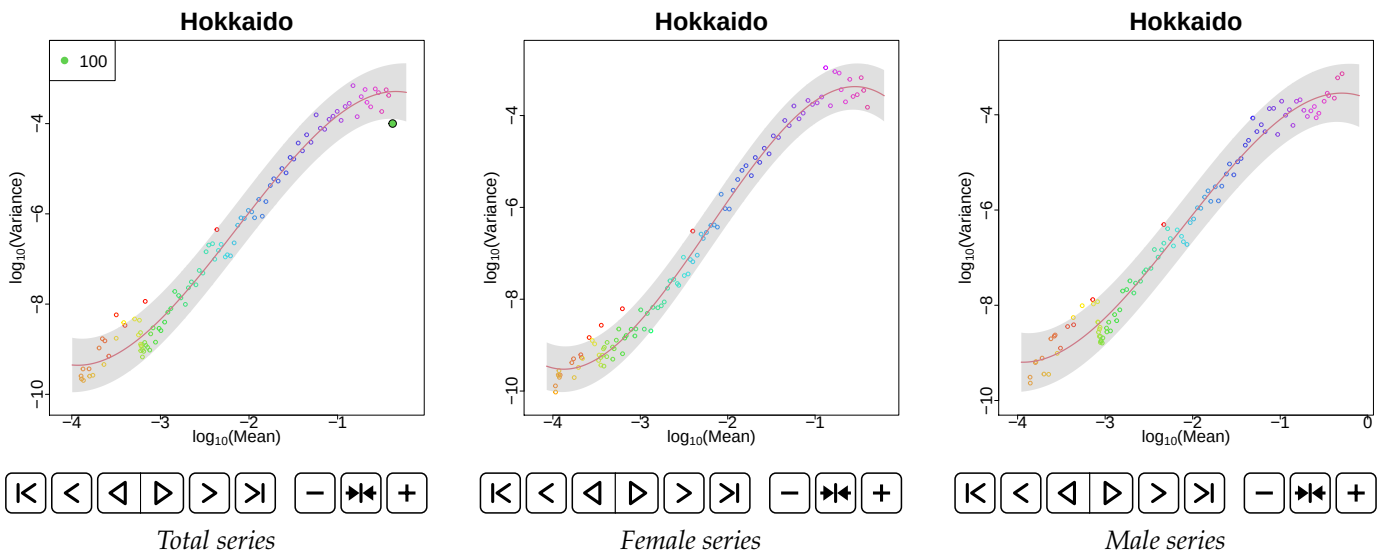

Figure A2: *Temporal TL fitted models with 95% prediction intervals for Japanese age-specific mortality rates by prefecture. Solid dots filled with colors represent outlying observations detected using the Mahalanobis distance.*

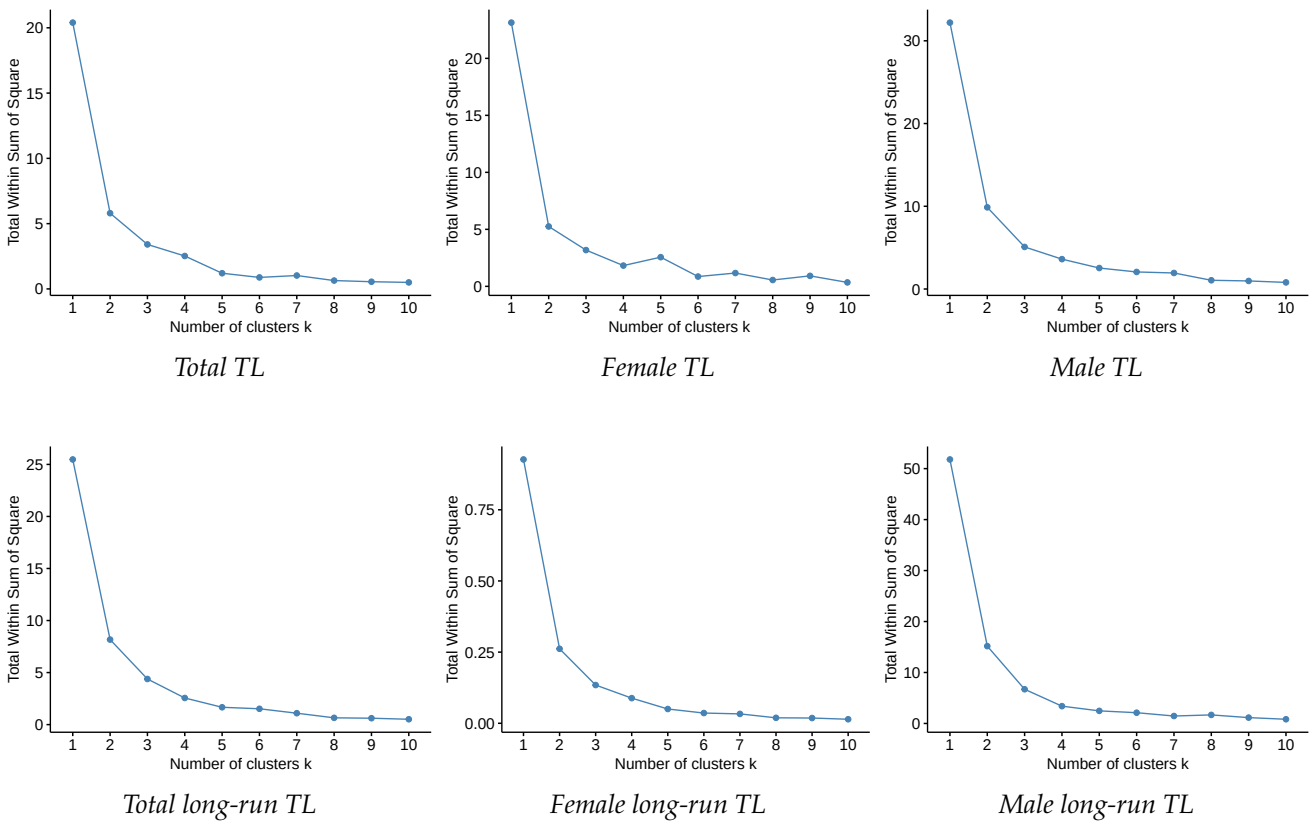

Figure A3: *Within-cluster sum of squares for K-means clustering applied to the temporal TL slope estimates.*

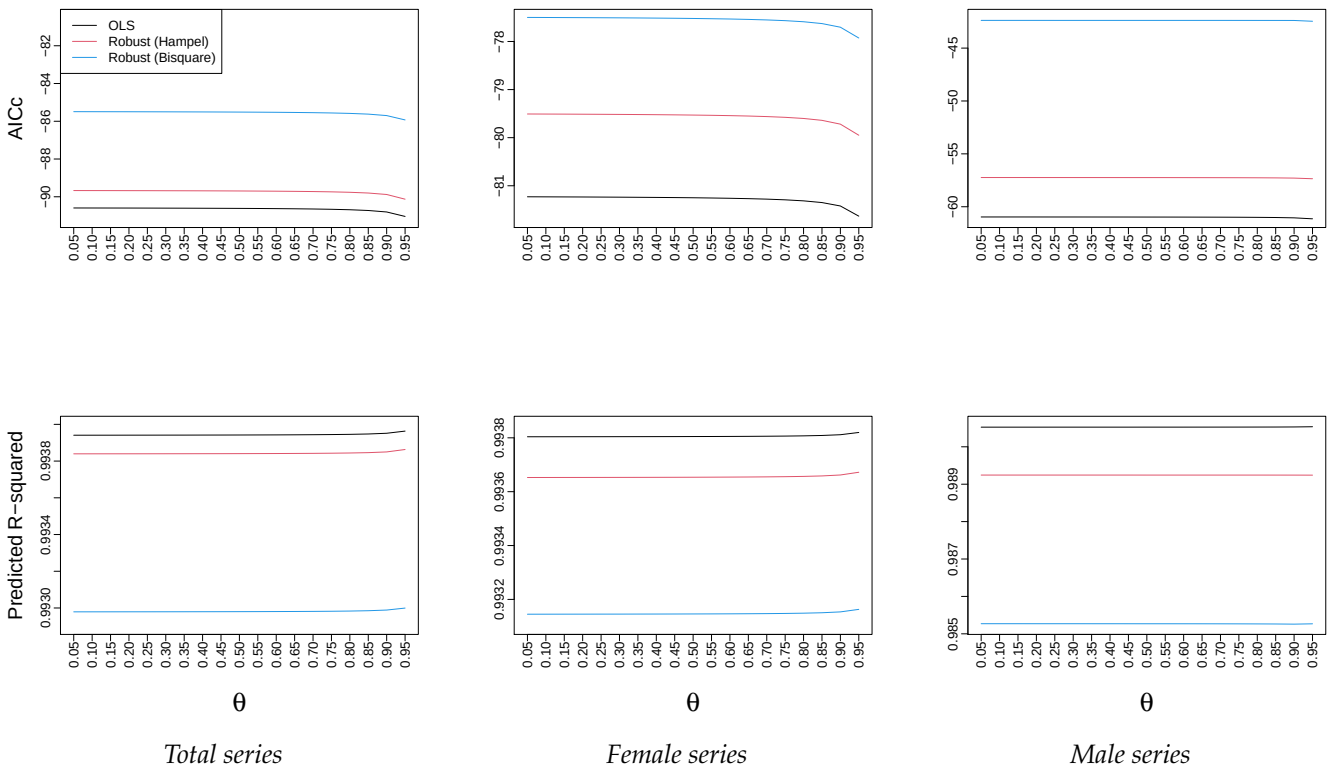

Figure A4: *AICc and predicted R^2 or total, female and male mortality series as functions of θ.*