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Formal Relationship 23

Is the fraction of people ever born who are currently alive rising or falling?

Joel E. Cohen

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Joel E. Cohen [1]

Abstract

BACKGROUND
Some journalists and demographers have asked: How many people have ever been born? What is the fraction $F(t)$ of those ever born up to calendar year t who are alive at t? The conditions under which $F(t)$ rises or falls appear never to have been analyzed.

OBJECTIVE
We determine under what conditions $F(t)$ rises or falls.

METHODS
We analyze this question in the model-free context of current vital statistics and demographic estimates and in the context of several demographic models.

RESULTS
At present $F(t)$ is very probably increasing. Stationary, declining, and exponentially growing population models are incapable of increasing $F(t)$, but a doomsday model and a super-exponential model generate both increasing and decreasing $F(t)$.

CONCLUSIONS
If the world's human population reaches stationarity or declines (which many people expect to happen within a century), the presently rising fraction of people ever born who are now alive will then be falling.

COMMENTS
It is curious that nearly all empirical estimates of the number of people ever born assume exponential population growth, which cannot explain increasing $F(t)$.

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1. Introduction

Journalists and some estimable demographers have asked: How many people have ever been born? What is the fraction $F(t)$ of those ever born up to calendar year t who are alive at t? After reviewing answers to these questions, Tattersall (1996:335) estimated that $F(1973) \approx 0.04 < F(1992) \approx 0.067$.

Here we answer a theoretical question apparently not addressed previously: Under what conditions does $F(t)$ rise or fall? We analyze this question in the model-free context of current vital statistics and demographic estimates and in the context of several demographic models. We infer, based on empirical estimates of others, that at present $F(t)$ is very probably increasing but that it must decrease if the human population size approaches stationarity or declines. Stationary, declining, and exponentially growing population models are incapable of increasing $F(t)$, but a doomsday model and super-exponential model can generate increasing and decreasing $F(t)$.

2. Theoretical analysis in discrete time

We consider a population closed to migration, e.g., the global human population. Whether $F(t)$ is rising or falling at time t depends on the population alive $A(t)$ at time t, the cumulative number of births $B(t)$ prior to and including t ("the number of people ever born"), the number of births $b(t)$ in the coming unit of time (here a year), and the number of deaths $d(t)$ in the coming year. We assume all four quantities $A(t)$, $B(t)$, $b(t)$, and $d(t)$ are strictly positive. For mnemonic convenience, A = Alive (now), B = Born (ever). Then, in discrete time, $F(t)$ rises from time t to time $t+1$ (e.g., from January 1, 2013 to January 1, 2014) if and only if

$$\begin{aligned} F(t) = A(t)/B(t) &< F(t+1) \\ &= A(t+1)/B(t+1) \\ &= [A(t)+b(t)-d(t)]/[B(t)+b(t)]. \end{aligned} \tag{1}$$

By elementary algebra, the inequality (1) is true if and only if

$$[b(t)-d(t)]/A(t) > b(t)/B(t). \tag{2}$$

Condition (2) is intuitive. It asserts that the fraction $F(t)$ of those ever born who are now alive rises from t to $t+1$ if and only if the fractional change over the next year in the number alive (which is the left side of (2)) exceeds the fractional change over the next year in the number ever born (which is the right side of (2)). Because the right side of (2) is positive, the left side must also be positive, so a necessary consequence of (1) is that $d(t) < b(t)$, i.e., the population increases from time t to $t + 1$. Under this condition, (2) is algebraically equivalent to

$$A(t)b(t)/[b(t) - d(t)] < B(t). \tag{3}$$

2.1 Empirical example: $F(t)$ is currently increasing

The quantities on the left side of (3), namely $A(t)$, $b(t)$, and $d(t)$, are estimated by international statistical agencies. By contrast, estimates of $B(t)$ necessarily involve conjectures about human demographic prehistory and the ill-defined question of which human ancestors count as humans. We shall use the left side of (3) to determine how big $B(t)$ must be if $F(t)$ is to be increasing. We shall then compare this lower bound (the left side of (3)) with several published estimates of $B(t)$.

For the world in calendar year 2013, $b(t) = 134,127,000$, $d(t) = 56,083,000$ (U.S. Census 2013). The mid-year population in 2012 was estimated at 7,017,544,000 and the mid-year population in 2013 was 7,095,218,000 (U.S. Census 2013). The seven significant figures in these estimates are spuriously precise, as most censuses have errors of at least a few percent and many regions have no recent censuses or vital statistical systems. Nevertheless, for this example, I retain the precision of the Census estimates. I estimated the world's population on January 1, 2013 as the geometric mean of the Census's midyear populations for 2012 and 2013, namely, $A(t) = 7,056,274,000$. With these estimates, $A(t)b(t)/[b(t) - d(t)] = 12,126,965,000$, about 12.1 billion.

Prior estimates of $B(t)$, the number of people ever born by year t, have varied from a low of 20.32 billion (Petty 1682) to a high of 5.3 trillion (Winkler 1959) (Table 1). This uncertainty by more than two orders of magnitude seems appropriate for an estimate based on so many unknowns. No author in Table 1 recognized an uncertainty in his estimate of $B(t)$ commensurate with the variation of estimates among authors. The lowest of these estimates exceeds the threshold value of $B(t)$, about 12.1 billion, required for $F(t)$ to be increasing. A currently plausible range for $B(t)$, perhaps 50 billion to 150 billion, greatly exceeds the threshold. I conclude that the current fraction $F(t)$ of people ever born who are now alive is very likely to be rising.

The United Nations Population Division (2013) estimated that the total number of births in the world from 1960 through 2014 is 7.04 billion (in the medium variant projection). If the low and high estimates (excluding Petty's) in Table 1 are increased by 7 billion to take into account births since those estimates were published, then approximately $0.1\% \leq F(2013) \leq 9.3\%$.

Table 1: Estimates of $B(t)$, the number of people ever born by year t

Year t	**People ever born $B(t)$ (billion = 10^9) by t**	**Source**
1682	20.32	Petty 2004 (1682), Postscript, paragraph 2
1760	120	Ezra Stiles, in Tattersall 1996:331
1959	$3,390 - 5,260$	Winkler 1959:75
1960	69	Keyfitz 1966:581
1960	110	Deevey 1960:197
1962	$77 - 96$	Desmond 1962, as reprinted 1965:21
1992	79.6	Tattersall 1996:335
2002	106	Haub 2002

3. Theoretical analysis in continuous time

3.1 Stationary or declining population

In a stationary population, $F(t) = A(t)/B(t)$ falls because $A(t)$, the number of people alive, is constant in time while $B(t)$, the number of people ever born up to time t, increases with time if there are any births and deaths. In a closed population in which deaths at t outnumber births at t, $F(t)$ falls because $A(t)$ decreases in time while $B(t)$ increases with time if births occur and remains constant if there are no births. If the world's human population reaches stationarity or declines (which many people expect to happen within a century), the presently rising fraction of people ever born who are now alive will then be falling.

3.2 Continuous time in general

Denote the derivative with respect to time by $'$. Let $b(t) > 0$ be the continuous density of the number of births per unit of time (not *per capita*, but in total) at time t. Let $d(t) > 0$ be the continuous density of the number of deaths per unit of time (not *per capita*, but in total) at time t. Let $B(t) = \int_0^t b(s)ds$ be the cumulative number of births from 0 to t, so that the derivative with respect to time is $B'(t) = b(t)$. We assume now, and for the remainder of this paper, that $t = 0$ is chosen sufficiently far in the past that the cumulative number of births prior to $t = 0$ is negligible compared to the number of births at and after $t \geq 0$. Under this assumption, we interpret $B(t)$ as the cumulative number of *all* births up to t. Let $D(t) = \int_0^t d(s)ds$ be the cumulative number of deaths from 0 to t, so that $D'(t) = d(t)$.

At $t > 0$, $A(t) = A(0) + B(t) - D(t)$ so $A'(t) = b(t) - d(t)$. Because $F(t) = A(t)/B(t)$,

$$\begin{aligned} F'(t) &= [B(t)A'(t) - B'(t)A(t)]/B^2(t) \\ &= [B(t)(b(t) - d(t)) - b(t)(A(0) + B(t) - D(t))]/B^2(t) \\ &= \{b(t)[D(t) - A(0)] - B(t)d(t)\}/B^2(t) \\ &= \{b(t)[B(t) - A(t)] - B(t)d(t)\}/B^2(t). \end{aligned}$$

Hence $F'(t) > 0$ if and only if the numerator of the last expression is positive, i.e., if and only if $d(t)/b(t) < 1 - A(t)/B(t)$ which is the same as (2) and (3). In (1) and (2), $b(t)$ and $d(t)$ are the numbers of births and deaths, respectively, over the time interval $[t, t+1)$. In the continuous-time model here, $b(t)$ and $d(t)$ are the continuous densities of births and deaths, respectively. Despite this difference of interpretation, the formulas that give the conditions for increasing $F(t)$ are the same.

3.3 Exponential model

Consider a population with no age structure that starts with $A(0)$ individuals and has a time-independent birth rate $\beta > 0$ *per capita* and a time-independent death rate $\delta > 0$ *per capita*. The rate of change of population size *per capita* is $r_{ex} = \beta - \delta$. Having considered stationary populations above, I assume here that $r_{ex} \neq 0$. Then

(4) $$\frac{1}{A(t)}\frac{dA(t)}{dt} = r_{ex}.$$

The population size $A(t) = A(0)\exp(r_{ex}t)$ and the instantaneous densities per unit time of births $b(t) = \beta A(t) = \beta A(0)\exp(r_{ex}t)$ and deaths $d(t) = \delta A(0)\exp(r_{ex}t)$ all change exponentially at the same exponential rate. The number of people ever born by time $t > 0$ (still ignoring births before $t = 0$) is the integral of $b(t) = \beta A(t)$ from 0 to t, namely, $B(t) = (A(0)\beta/r_{ex})(\exp(r_{ex}t) - 1)$. The fraction $F(t)$ of people ever born who are alive at t always decreases because$F'(t) = -r^2\exp(rt)/(b[\exp(rt) - 1]^2)$. If $\delta < \beta$, then $F(t)$ approaches a positive limit:

$F(t) = A(t)/B(t) = (r_{ex}/\beta)/[1 - exp(-r_{ex}t)] \to r_{ex}/\beta = 1 - \delta/\beta > 0$ as $t \to \infty$.

The exponential model cannot account for the empirical observation in section 2.1 that $F(t)$ is now increasing with increasing time. In light of this observation, it is curious that nearly all published empirical estimates of the number $B(t)$ of people ever born assumed exponential or piecewise exponential population growth.

3.4 Doomsday model

Von Foerster, Mora, and Amiot (1960) analyzed a "doomsday" model in which the growth rate *per capita* of the population size $A(t)$ is proportional to population size. A qualitative difference between the exponential model with $r_{ex} > 0$ and the doomsday model with $r_{dd} > 0$ is that the exponential model takes infinite time to approach infinite population size, whereas the population size in the doomsday model diverges to infinity in finite time $t_\infty \equiv 1/(A(0)r_{dd})$. The doomsday model assumes that, for some constant $r_{dd} > 0$,

$$\frac{1}{A(t)}\frac{dA(t)}{dt} = r_{dd}A(t) \quad \text{or} \quad \frac{dA(t)}{dt} = r_{dd}(A(t))^2.$$

Then for $0 \leq t < t_\infty$,

$$\begin{aligned}
A(t) &= \frac{A(0)}{1 - A(0)r_{dd}t} \uparrow \infty \quad \text{as} \quad t \uparrow t_\infty, \\
B(t) &= \beta \int_0^t A(s)ds = -(\frac{\beta}{r_{dd}}) \log(1 - A(0)r_{dd}t), \\
F'(t) &= -(\frac{1}{\beta}) \left(\frac{A(0)r_{dd}}{(1 - A(0)r_{dd}t) \log(1 - A(0)r_{dd}t)} \right)^2 (1 + \log(1 - A(0)r_{dd}t)).
\end{aligned}$$

It follows that $F'(t) < 0$ if and only if $-(1 + \log(1 - A(0)r_{dd}t)) < 0$. As $A(0)r_{dd} = 1/t_\infty$, we have $F'(t) < 0$ if and only if $t/t_\infty < 1 - 1/e$. Consequently, for the first $1 - 1/e \approx 63\%$ of the population's lifetime from 0 to t_∞ , the fraction of people ever born who are currently alive decreases. For the final $1/e \approx 37\%$ of the population's lifetime, the fraction of people ever born who are currently alive increases.

This model is a special case of a "Malthus-Condorcet" model in which the rate of increase of human carrying capacity of Earth is proportional to the rate of increase of the human population (Cohen 1995).

3.5 Super-exponential model

The exponential model and the doomsday model are special cases of a model of super-exponential growth. Suppose that the population size $A(t)$ at time t satisfies

$$\frac{1}{A(t)}\frac{dA(t)}{dt} = r(A(t))^{\varepsilon}, \qquad \varepsilon \geq 0$$

where r and ε are assumed constant in time. This model becomes the stationary model when $r = 0$. As the stationary model was analyzed above, we assume henceforth that $r > 0$. This model becomes the doomsday model (with $r = r_{dd}$) when $\varepsilon = 1$ and the exponential model (with $r = r_{ex}$) when $\varepsilon = 0$. As the exponential model was analyzed above, we assume henceforth that $\varepsilon > 0$. As the doomsday model was analyzed above, we also assume henceforth that $\varepsilon < 1$. Then this super-exponential model also approaches an infinite population size in finite time: for $0 \leq t < t_{\infty} \equiv 1/(\varepsilon r A(0)^{+\varepsilon})$, population size $A(t)$ is

$$A(t) = \frac{1}{(A(0)^{-\varepsilon} - \varepsilon r t)^{1/\varepsilon}} \uparrow \infty \quad \text{as} \quad t \uparrow t_{\infty}.$$

Assume a time-independent birth rate β *per capita* so that $b(t) = \beta A(t)$ and

$$B(t) = \beta \int_0^t A(s)ds = \beta/(r(1-\varepsilon)) \cdot [(A(0)^{-\varepsilon} - \varepsilon r t)^{1-1/\varepsilon} - A(0)^{1-\varepsilon}].$$

Then

$$\begin{aligned} F(t) &= A(t)/B(t) = \frac{A(0)^{\varepsilon}(\varepsilon - 1)r)}{\beta[(1 - A(0)^{\varepsilon}\varepsilon r t)^{1/\varepsilon} + A(0)^{\varepsilon}\varepsilon r t - 1]}, \\ F'(t) &= -\frac{(A(0)^{\varepsilon} r)^2(\varepsilon - 1)}{\beta[(1 - A(0)^{\varepsilon}\varepsilon r t)^{1/\varepsilon} + A(0)^{\varepsilon}\varepsilon r t - 1]^2}(\varepsilon - (1 - A(0)^{\varepsilon}\varepsilon r t)^{-1+1/\varepsilon}). \end{aligned}$$

By continuity between the super-exponential model ($0 < \varepsilon < 1$) and the doomsday model ($\varepsilon = 1$), it is obvious without calculation that for some parameter values $F(t)$ will be decreasing at some early times and increasing at some later times. Precisely,

$$F'(t) < 0 \quad \text{if and only if} \quad \frac{t}{t_{\infty}} < 1 - \varepsilon^{\varepsilon/(1-\varepsilon)}, \quad \text{and}$$

$$F'(t) > 0 \quad \text{if and only if} \quad \frac{t}{t_{\infty}} > 1 - \varepsilon^{\varepsilon/(1-\varepsilon)}.$$

As $\varepsilon \to 1$, $\varepsilon^{\varepsilon/(1-\varepsilon)} \to \frac{1}{e}$, giving the result for the doomsday model.

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