BOOK REVIEW

Marius Iosifescu, *Finite Markov Processes and Their Applications*, John Wiley and Sons, New York, 1980, 295 pp., $35.95.

A Markov process is a mathematical abstraction created to describe sequences of observations of the real world when the observations have, or may be supposed to have, this property: only the most recent observation, and not any earlier observation, affects the probability distribution of future observations. This property is known as the Markov property. It is sometimes stated this way: given the present, the past and the future are conditionally independent. A Markov process is the random analog of a first-order ordinary differential equation; as a model of a deterministic system, such a differential equation specifies that the future change depends at most on the present state of the system and not on its past.

The Markov property does describe enough real observations well, and is sufficiently tractable to mathematics, that the theory and applications of Markov processes have proliferated enormously since Markov introduced such processes in 1906. It is safe to say that no biologist who analyzes quantitatively time series with randomness can read the research in his or her field without some understanding of Markov processes. A grasp of Markov processes is required of applied mathematicians interested in stochastic phenomena in biology.

The excellent new text by Iosifescu, a revised edition of a book published in Rumanian in 1977, offers a great deal both to the beginner and to the expert in Markov chains.

For the beginner, the book starts with a review of the needed elements of probability theory and linear algebra. This review gives the beginner simple, concrete examples, e.g., of a 2×3 matrix and its transpose, to illustrate the notion of a transposed matrix. It also reminds the expert of the Daniell Kolmogorov theorem, which proves the existence of infinite sequences of independent random variables that have consistent joint probability distributions.

Chapter 2 introduces the fundamental concepts of homogeneous Markov chains (in discrete time) as well as nine concrete examples of Markov processes. These examples are analyzed in greater detail later in the book. There is a precise and intelligible account of stopping times and the strong Markov property.

Absorbing Markov chains are analyzed using the fundamental matrix along the lines laid down by J. G. Kemeny and J. L. Snell in their 1960 classic, *Finite Markov Chains*. Iosifescu adds an account of the conditional transient behavior of absorbing chains, a problem identified by M. S. Bartlett and solved by J. N.

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Darroch and E. Seneta. Briefly, the problem is to give conditions under which there exists a limiting probability that the chain is in a given transient state, conditional on its not yet having entered any absorbing state; and when such a limit exists, to calculate it. I have not seen these elegant and important results in book form before.

In discussing ergodic Markov chains, Iosifescu again uses the fundamental matrix of Kemeny and Snell. He adds an account of the two-particle method due to W. Doeblin and exploited by D. Griffeath. Among the remarks in this chapter that were new to me is the observation that, for a regular Markov chain, the mean number of visits to state $j \neq i$ between two occurrences of state i equals $\pi(j)/\pi(i)$, where $\pi(i)$ is the long-run or equilibrium probability of occurrence of state i.

Iosifescu reviews the geometric interpretation, due to N. Pullman and R. Z. Yeh, of recurrent and transient states; the relation between the tail sigma-field of a chain and the classification of states, due principally to Blackwell and Freedman in the homogeneous case; and the limit theorems for partial sums of a real-valued functional of a Markov chain. Here, R. L. Dobrushin's theorem describes the bizarre possible behavior of functionals of a two-state Markov chain.

An important theorem published by O. Onicescu and G. Mihoc in 1940 gives the expected value of the characteristic function of partial sums of real-valued functionals of a Markov chain. In modern language, their formula would be called a discrete-time Feynman-Kac formula for a finite Markov chain. Their formula was independently rediscovered and published by the French demographer Hervé LeBras in 1974, and by me in 1979; and how many other times, I do not know. Perhaps publication of the theorem of Onicescu and Mihoc in this book will diminish the rate of rediscovery.

One chapter is devoted to mathematical learning theory and population genetics. Iosifescu points out that the models in the famous 1955 book by R. R. Bush and C. F. Mosteller, *Stochastic Models for Learning*, are chains with complete connections or generalizations of them, though the American authors were unaware of the invention and analysis of these processes 20 years earlier by Onicescu and Mihoc. This chapter shows that Iosifescu took the applications seriously enough to learn more than just the mathematics involved.

The topic of nonhomogeneous chains does not appear in the book of Kemeny and Snell. In *Markov Chains: Theory and Applications*, D. L. Isaacson and R. W. Madsen (1976) describe ergodicity coefficients and weak and strong ergodicity. Iosifescu's chapter on nonhomogeneous chains goes considerably further. He observes that the strong Markov property holds for a nonhomogeneous chain, but, very surprisingly (as U. Krengel showed), if t is a stopping time and X the Markov chain, then $X(n+t)$, $n = 0, 1, 2, \ldots$, need not be a Markov chain, unlike the homogeneous case. He also gives H. Cohn's characterization of the tail sigma-field of a nonhomogeneous chain and Dobrushin's central limit theorem for such chains.

Iosifescu introduces the final chapter of the book, devoted to Markov processes, thus: "the finite setting allows a reasonably elementary treatment which avoids the usual intricacies that lead one to ask whether the present theory of stochastic processes is not more difficult than its applications warrant." I like this spirit. In spite of it, even in the finite case, some results are so difficult to prove that Iosifescu omits the proofs. He gives the Feller representation of the transition probability function, which is not usually found in elementary accounts. He also gives J. F. C. Kingman's theorem on the relation between discrete skeletons of a process and their continuous limits.

Every chapter has exercises. These state additional results that are as interesting as those in the main text.

One mission of the book, as Iosifescu explains in some historical notes, is "to stress the importance of the contributions to the theory of finite Markov chains and their generalizations made by the founders of the Romanian probability school, Octav Onicescu and Gheorghe Mihoc." This mission is accomplished.

I expect this book to be a useful text and valuable reference book for finite Markov chain theory and application for years to come. While I wish the index were more systematic, the text is very clearly written and precise without being fussy. It has concrete examples in profusion and theoretical results that are not, so far as I know, available in any other book.

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