ABSTRACTS OF PRESENTED PAPERS

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184-27. The Stability of Large Random Systems. Joel E. Cohen, Rockefeller University and Charles M. Newman, University of Arizona.

Let A(1), A(2), . . . be a sequence of i.i.d. random real n x n matrices. Let $x(t) = A(t)x(t-1)$, $t = 1, 2, \ldots$, with $0 \neq x(0) \in R^n$. Define $\log \lambda_n(x(0)) = \lim_{t\uparrow\infty} t^{-1} \log ||x(t)||$, where $||\cdot||$ denotes, e.g. the Euclidean norm, providing the limit exists a.s. Say that the product of random matrices is strongly stable (respectively unstable) is $\sup\{\log \lambda_n(x(0)): 0 \neq x(0) \in R^n\} < 0$ (resp., $\inf\{\log \lambda_n(x(0)): 0 \neq x(0) \in R^n\} > 0$) a.s. If, for each n, the entries $A_{ij}(t)$ are i.i.d. mean zero, variance s^2, then $\lim\sup ns^2 < 1$ (resp. $\lim\inf ns^2 > 1$) implies strong stability (resp. instability) for large n with no further assumptions (resp. with some further technical assumptions). If in addition the $A_{ij}(t)$'s are normal, then $\log \lambda_n(x(0)) = [\log s^2 + \log 2 + \Psi(n/2)]$ (independent of $x(0)$), where Ψ is the digamma function. These conditions for the asymptotic stability or instability of the product of random matrices are of the form proposed by May. Counterexamples show that May's (NATURE 238(1972) 413-414) criteria are not valid in general for the system of linear ordinary differential equations that he originally considered, nor for the related system of difference equations considered by Hastings (J. THEOR. BIOL. 97(1982) 155-166, BULL. AMER. MATH. SOC. 7(1982) 387-388).
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