181-34. Eigenvalue Inequalities for Random Evolutions, Joel E. Cohen, Rockefeller University and Center for Advanced Study in the Behavioral Sciences, Stanford.

A random evolution is a model for a population whose growth is piecewise exponential, with a random growth rate that is a functional of a continuous-time discrete-state Markov chain. Suppose the population experiences $n \geq 2$ different environments, and grows exponentially at rate b_i in the i^{th} environment, $i = 1,\ldots,n$. Suppose the succession of environments is determined by a continuous-time Markov chain with $n \times n$ infinitesimal generator A and $N(t)$ is the positive scalar size of the population at time t. Suppose $dN(t)/dt = b(t)N(t)$, $N(0) = 1$, where $b(0) = b_1$ say and $P[b(s+t) = b_j | b(s) = b_i] = (e^{At})_{ij}$ and $\{b_1,\ldots,b_n\}$ is a set of n finite real numbers. If A is irreducible and $B = \mathrm{diag}\,(b_1,\ldots,b_n)$, then $(1/t) \log E(N(t)) \to \log\ r(e^{A+B})$ as $t \to \infty$, where r = spectral radius. An upper bound is obtained on $r(e^{A+B})$ as one of several new inequalities of the form (*) $f(e^A e^B) \geq f(e^{A+B})$, where A and B are $n \times n$ matrices of complex numbers and f is a real-valued continuous function of a matrix argument that is finite when all elements of the matrix are finite. Such inequalities (*) have arisen independently in statistical mechanics (Received April 12, 1982)