

181-9. The Invariant Distribution of Markovian Products of Random Matrices. Joel E. Cohen,

Rockefeller University & Center for Advanced Study in the Behavioral Sciences, Stanford.

Consider a sequence of vectors . . . $Y(t-1), Y(t), Y(t+1), \dots$, with nonnegative real elements, such that $Y(t) = X(t)Y(t-1)$, where $X(t)$ is an $n \times n$ nonnegative matrix and the sequence of matrices . . . $X(t-1), X(t), X(t+1), \dots$ is a sample path of a discrete-time Markov chain \underline{X} whose state space is a (nice) set of nonnegative $n \times n$ matrices. (This model arises naturally in population dynamics, among other applied fields.) Let $y(t) = Y(t) / \|Y(t)\|$, for some norm $\|\cdot\|$. Then $(y(t), X(t))$ is Markovian, though $y(t)$ by itself is not. Given the transition probability function of the $X(t)$ process, one can write explicitly the transition probability function of the $(y(t), X(t))$ process. If \underline{X} is suitably ergodic, then $(y(t), X(t))$ becomes independent of $(y(t-s), X(t-s))$ asymptotically as s gets large. When the Markov process \underline{X} is time-homogeneous as well as sufficiently ergodic, the distribution of $(y(t), X(t))$ converges to a distribution that can be calculated. From this limiting distribution, one can also calculate the asymptotic almost sure rate at which $\underline{Y}(t)$ changes in size, known as the Furstenberg-Kesten limit of the Liapunov characteristic. A numerical example will be given. (Received March 8, 1982)