Book Reviews

Reversibility and Stochastic Networks
Frank P. Kelly — *Joel E. Cohen*

Reversibility and Stochastic Networks, by Frank P. Kelly, John Wiley, 1979, viii + 230 pp., $39.95.

This superb book displays the power and beauty of combining two simple ideas. The two ideas are reversibility and vector-valued Markov processes. A stochastic process $X(t)$ is reversible if $(X(t_1), \ldots, X(t_n))$ has the same distribution as $(X(\tau - t_1), \ldots, X(\tau - t_n))$ for all positive integers n and all possible times $t_1, \ldots, t_n$ and τ. A stochastic process $X(t)$ is a Markov process, in the sense usually used in this book, if it is a continuous-time, homogeneous, irreducible, aperiodic Markov chain with a countable set $\mathscr{S}$ of states $x = (x_1, \ldots, x_J)$. Usually x_i is a nonnegative integer that specifies some attribute of the ith component of the process.

Since a reversible stochastic process is stationary, a reversible Markov process must be in equilibrium. Reversibility, and its near relatives quasi-reversibility and dynamic reversibility, make it possible to calculate in closed form the equilibrium probability distributions of Markov processes whose complexity would otherwise appear hopelessly forbidding. In many cases, these equilibrium probability distributions can even be proved valid, using the method of stages, for non-Markovian processes of the same structure but with nonexponential state lifetimes.

The simplest example of the power of reversibility arises when $\mathscr{S}$ is just some integers. If $q(j, k)$ is the probability flux from state j to state $k = j$, and $q(j, j) = 0$, the equations for the equilibrium probability distributed $\pi(k) \geq 0$, $\sum_k \pi(k) = 1$, in the general case

$$\pi(j) \sum_k q(j, k) = \sum_k \pi(k) q(k, j), \quad \text{for all } j,$$

reduce in the reversible case to the detailed balance conditions,

$$\pi(j) q(j, k) = \pi(k) q(k, j), \quad \text{for all } j, k,$$

which are manifestly easier to solve.

A subtler example gives the flavor of the more elaborate uses of reversibility in this book. An $M/M/1$ queue is a queue with customers arriving in a Poisson stream and a single server whose service times are independently exponentially distributed for each customer. Surprising fact: the number $n(t)$ of customers in an $M/M/1$ queue at time t is independent of the departure process from the queue prior to time t. Why? $n(-t)$ and the arrival process after $-t$ are obviously independent. Since an $M/M/1$ queue is reversible, $n(-t)$ and the arrival process after $-t$ have the same joint distribution as $n(t)$ and the departure process prior to t.

Kelly progresses in easy, intelligible steps through an enormous range of models. He starts from birth and death processes and simple queues. He uses vector-valued processes to describe series of simple queues, open and closed networks of queues with paths depending on customer type and arrival rates depending on system state (to mention just two of the many complications treated analytically), flow models, invasion models, compartmental models, migration processes, clustering processes, spatial processes, and Markov random fields. He weaves general theorems together with concrete applications that lead to explicit formulas.

Among the concrete applications are the infinite neutral allele model of population genetics, communication networks and telephone switching systems, interspecific competition, time-sharing computers, electronic counters, quality control inspection along a conveyor belt, electrical networks, epidemics among plants, mechanics in a garage, maintenance in a trucking firm, organizational hierarchies and manpower systems, mining operations, road traffic, social grouping behavior, and polymerization.

Kelly illuminates his material with the understanding and care of an old jeweler examining a diamond. He shows that one stochastic process can model a diversity of real activities, and that one real activity can be modeled by a diversity of stochastic processes. He even mentions that, in real applications, it is necessary to check whether the assumptions of a model are met. Topics once presented reappear as new theorems and throughout the book are comments that convey, and reveal on the part of the author, a wonderful insight into how all parts of the book are related. This is the instruction one would hope for in a private tutorial from the author.

The mathematics is healthy mathematics. It is the normal physiology of Markov processes that remain in each state for a positive length of time, rather than the morbid pathology of processes that can pass through an infinite number of states in a finite time. The notation is always natural, uncluttered, and helpful. A complete symbol index gives one-line reminders of definitions in addition to page references. Part of the reason the mathematics is so clear is that Kelly's pleasant, simple English exploits fully the power of natural language to describe abstract processes.

Of the 142 publications cited only 57 date from before 1970; so much of this material has probably not appeared in a book before. The 277 thoughtful problems (two of them marked "hard") present some published results and, I would guess, many new observations of Kelly's. I guess so because, following one section based on (and extending) a paper of mine, the problems present facts about my model that were new to me!

What more could one ask of this outstanding book? As a user of stochastic processes for biological modeling, I am always looking for guidance on how to estimate the parameters and test the appropriateness of models. Here I found only three scattered references to sufficient statistics. Perhaps the statistical methods for these models belong in a separate book. If so, I would be delighted if Kelly would write another book as coherently conceived and as beautifully executed as this one.

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