Conference on Spatial Processes

Newcastle upon Tyne, 20-21 May 1974

Conference on Stochastic Models in Space and Time

Edinburgh, 19-20 December 1974

Reprinted from the
ADVANCES IN APPLIED PROBABILITY
Vol. 7, No. 3, pp. 449–467
September 1975

Stochastic ergodicity of population age structure

JOEL E. COHEN, *Kings College Research Centre, Cambridge*

This paper (Cohen (1975)) establishes a new class of ergodicity theorems for the age structure of populations, and proposes applications.

Let $S = \{A_1, A_2, \cdots\}$ be a denumerable set of Leslie matrices A_i satisfying the assumptions of the Coale-Lopez theorem of weak ergodicity for populations, and let $\boldsymbol{m}(0)$, $\boldsymbol{n}(0)$ be any two initial population age structures (column vectors) satisfying the assumptions of the Coale-Lopez theorem. Suppose $\boldsymbol{m}(t)$ is the population age structure obtained at time t by premultiplying $\boldsymbol{m}(0)$ sequentially by t elements of S, $A_1, \cdots$ where the sequence of matrices following A_1 is determined by a homogeneous irreducible aperiodic positive recurrent geometrically convergent Markov chain on the state space S; and suppose that $\boldsymbol{n}(t)$ is independently determined by the same Markov chain in the same way, starting from A_2. Then (*strong stochastic ergodicity*) all moments of the two random variables $m_i(t)/m_i(t)$ and $n_i(t)$ and $n_j(t)/n_i(t)$ converge and they converge in distribution as $t \to \infty$. The same conclusion holds if (*weak stochastic ergodicity*) the Markov chain is finite and weakly ergodic (in the sense of Hajnal), but not necesearily homogeneous.

References

Cohen, J. E. (1975) Ergodicity of age structure in populations with Markovian vital rates. (1) Countable states. (To appear).

Lopez, A. (1961) *Problems in Stable Population Theory:* Office of Population Research, Princeton.

Hajnal, J. (1958) Weak ergodicity in non-homogeneous Markov chains. *Proc. Camb. Phil. Soc.* **54,** 233–245.