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CHILDHOOD MORTALITY, FAMILY SIZE AND BIRTH ORDER IN PRE-INDUSTRIAL EUROPE

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Abstract—Based on parish registers, demographic histories of Crulai (France), Tourouvre-au-Perche (France), and Geneva (Switzerland) established the childhood mortality experienced by complete sibships during periods of at least half a century before the French revolution. These observations may be presented as frequencies in incomplete five-dimensional contingency tables. The five dimensions are: survival (living or dead), completed sibship size, birth order, type of family (according to completeness of information about family), and epoch (period in which the family lived).

This paper reanalyzes these published data, using hierarchical log-linear models to discern which interactions among the five variables can justifiably be inferred from the data. The neonatal and infant mortality rates of firstborn are probably higher than those of later sibs (in Crulai and Tourouvre). But mortality by age 20 (in Geneva) is associated strongly with the epoch, type of family, and family size, and not significantly with birth order. The increase in mortality with completed family size is insufficient to select, in an evolutionary sense, for limited family size.

1. Introduction

This paper reanalyzes published data which describe the survival or the failure to survive to given ages of children in human families in Europe between 1550 and 1900. The purpose is to put inferences from the data about the relations among completed family size, birth order, childhood mortality, and parental cohort mortality on a more explicit statistical footing. The analysis is also intended to illustrate the power of log-linear models of incomplete multidimensional contingency tables and to emphasize with concrete examples needs for further theoretical development.

Many studies relating birth order to other variables are reviewed by Altus (1966) and Kammeyer (1967). Methodological problems are discussed by MacMahon, Pugh, and Ipsen (1960), Mantel and Halperin (1963), Barker and Record (1967), Haldane and Smith (1947), James (1969), and Veevers (1973).

Eighteenth-century data from England (McKeown and Brown, 1955, p. 133) demonstrate an apparent decrease in survival with increasing birth order. But completed sibship sizes were not included in the original observations, so either birth order or completed sibship size may control mortality.

In modern Birmingham, England (Gibson and McKeown, 1952), children of birth orders 4 and later suffered more than twice the infant mortality of firstborn. Since this study included all and only births occurring in 1947, completed sibship size could not be considered.

The need to observe completed sibship

size in testing for the effect of birth order is clear from an example (Magaud and Henry, 1968): Suppose that, within a family of a given final size, survival is identical for all birth orders, but that the larger the family, the lower the survival. Then if individuals are sampled at random, there will be an apparent inverse relation between birth order and survival only because higher proportions of the later birth orders will come from larger families.

Magaud and Henry (1968, p. 890) cite a 1906 study of Statistique Général de la France which found this hypothetical possibility to be real: "While they were living without either brothers or sisters, the firstborn incurred risks of death dependent on the final *future* size of the family." Magaud and Henry conclude that there is no influence of birth order on infant mortality. But they use no statistical tests for the randomness of the observed fluctuations in death rates by birth order.

Section 2 of this paper describes the sources and characteristics of the data used. Section 3 reviews briefly the method and models used. Section 4 presents the results of applying the methods of section 3 to the data described in section 2. Section 5 summarizes the conclusions drawn from the data in a way that is intended to be comprehensible in the absence of sections 3 and 4 and compares these conclusions with those drawn by the original authors of the data. An Appendix provides methodological details.

2. The Data

Procedures for reconstructing demographic histories of local European populations are described by Fleury and Henry (1965) and Henry (1970).

A family is defined as initiated by the marriage of a couple and terminated by the death of one spouse, separation, or disappearance from observation of the couple. If the family terminates after the wife reaches age 45, hence after the (presumptive) end of her childbearing years, the family is termed "complete." If the family terminates before the wife reaches age 45 because of the death of one of the partners, separation, or divorce, the family is termed "incomplete." If the family is not known to belong to one or another of these categories, it is called "other."

Gautier and Henry (1958) study marriages in Crulai in Normandy (France). For marriages between 1688 and 1742 whose children did not disappear from observation before they reached one year of age, they report the numbers living or dead (hereafter to be referred to as variable 1), according to the total number of children born into the family, or family size (hereafter variable 2), and according to the birth order (hereafter variable 3) of each child. They pool complete, incomplete, and other families. For consistency of format with the data reported for Geneva (to be described below), in families of three or more children, I collapse all the intermediate categories of birth order into one category called "middle." Hence the three categories of birth order are first, middle, and last for families of size three or more. For families of size two, there are no observations of middle births. For families of size one, there are observations of first births only.

The numbers of children who survived or died by one month are reported along with the corresponding numbers for the same families for one year. In the following analysis I treat these survivorship figures separately as two sets of data: "Crulai 1 month" and "Crulai 1 year."

Charbonneau (1970) reconstitutes the families formed by marriages between 1670 and 1769 in Tourouvre-au-Perche (France). Again, I collapse the categories of birth order into first, middle, and last. Each observation is classified by type of family (variable 4) as complete or not complete. The data on survivorship to one month and to one year are here treated separately as the data sets "Tourouvre 1 month" and "Tourouvre 1 year."

Henry (1956) studies marriages among the bourgeoisie (the hereditary ruling class) of Geneva, from approximately 1550 to 1899. He reports the survival to age 20 (variable 1) of children by size of family (variable 2) and the three categories of birth order (variable 3) according to whether the families were complete, incomplete, or other (variable 4) and according to which of seven half centuries (variable 5) the father of the family was born in. Since Henry (1956) uses only three categories for birth order (first, intermediate, and last) in Geneva, there is no choice but to use that or an even grosser classification.

Table 1 summarizes the characteristics of the data. The sources of the data analyzed here do not tabulate survival by sex of the child. Hence it is impossible to learn from these data whether the secular decline in childhood mortality affects the sexes differently.

3. Methods and Models

One approach to the data considers survival as a dependent variable (or an effect or response variable). A second approach considers all variables other than chronological time as possibly interacting dependent or response variables.

Under the first approach, only the interaction between survival and the remaining variables, singly or jointly, is examined. This approach assumes that the variables other than survival may affect survival, but not conversely.

The second, scientifically more conservative approach seeks a minimal model to describe the data. This approach assumes that only time is a truly independent variable. The interactions among the remaining variables are of interest.

In the following analyses, the second approach is used first for all five sets of data. Then survival is treated as a response variable in the four sets of data (Geneva excluded) and in an incomplete way for the Genevan data. With these data, these two approaches yield no dif-

Table 1.—Characteristics of the Data

Variable Number (i)	Variable Name	Crulai: Number Cells (c_i)	Crulai: Range	Tourouvre-au-Perche: Number Cells (c_i)	Tourouvre-au-Perche: Range	Geneva: Number Cells (c_i)	Geneva: Range
1	Survival	2	living/dead	2	living/dead	2	living/dead
2	Family size	13	1,2,...,13	15	4,5,...,18	11	2,3,...,11,≥12
3	Birth order	3	first/intermediate/last	3	first/intermediate/last	3	first/intermediate/last
4	Type of family	1	complete and other	2	complete/other	3	complete/incomplete/other
5	Epoch	1	marriages from 1688 to 1742	1	marriages from 1670 to 1769	7	half-century of father's birth: before 1600 to 1850-1899
Total cells		78		180		1,386	
Total logical zeros		6		0		42	
Total families		287		351		561	

Note: Family size is the number of live-born siblings. Both family size and birth order omit fetal losses and stillbirths. In the original data from Crulai and Tourouvre-au-Perche, but not Geneva, survival is reported for each birth order separately.

Sources: For Crulai, Gautier and Henry, 1958, p. 269; for Tourouvre-au-Perche, Charbonneau, 1970, pp. 409-413; for Geneva, Henry, 1956, pp. 216-217.

ferences in substantive conclusions, and such small quantitative differences in the measures of the association of survival with other variables, that it is judged not worth completing the treatment of survival as a response variable for the Genevan data.

If survival were assumed to be a response variable, then the continuously variable infant mortality rates could be approached by analysis of variance. But the assumed independent normal distribution of errors in the response variable seems sufficiently implausible for the present data to exclude analysis of variance or, equivalently, regression analysis on first approach.

This paper organizes the data into a multidimensional contingency table. A multidimensional contingency table is a generalization of an ordinary two-way contingency table with rows and columns. In the cells of a two-way table are counts of frequencies of occurrence. For example, a two-way table might be formed from one of the present sets of data by considering only variable 1 (survival) and variable 2 (family size). If the first row of such a table showed the number of children living after one month and the second showed the number dead while each column were identified with a given size of family, such a table would be appropriate for studying the possible interaction between family size and mortality.

Now suppose that the data in this two-dimensional contingency table were subdivided into a set of two-dimensional tables according to the birth order (variable 3) of the children within the families. For example, one two-way table would show the relation between survival and family size among firstborn children; another table, among children of middle birth order; and a third, among last-born children. (Since there are no children of middle birth order in families of size two, the two-way table for middle birth order would contain "logical zeros" for the numbers of surviving and of dead children in families of size two.) The resulting set of three two-way contingency tables is called a three-dimensional contingency table.

If this three-dimensional table were then subdivided into two or more three-dimensional tables according to the completeness of information about the family (variable 4), the resulting set of three-dimensional tables could be viewed as a four-dimensional table; and this in turn could be refined by the epoch of observation (variable 5) to give a set of four-dimensional tables or a single five-dimensional table.

The two-way table has two *margins*: one, the list of row sums, and two, the list of column sums. In the present example, the row sums show the number of children who lived and the number who died, irrespective of family size. The shorthand which will be used for this margin is [1], because the margin describes the frequency distribution of variable 1. Similarly, [2] is the list of column sums showing the frequency distribution of live births by family size.

From the perspective of the five-dimensional table, the two-way table of survival versus family size is also a margin. This margin shows the joint frequency distribution of variables 1 and 2, irrespective of the remaining variables. It will be denoted by [1, 2]. For the Genevan data, this margin appears in Table 8.

The models used in this paper to generate expectations with which the observed data may be compared are called hierarchical log-linear models. These models generalize the model of independence in a two-way (two-dimensional) table. That model of independence says that the relative frequencies (or probabilities) of the cells in the joint distribution of variables 1 and 2 should be the product of the relative frequencies in the corresponding rows and columns of the margins. The shorthand which will be used for such a model is [1] [2] because the margin [1] is assumed to interact multiplicatively with the margin [2]. If the observed frequencies agree reasonably well (by some statistical criterion) with the expected frequencies of this model, then there are no statistical grounds for assuming that variables 1 and 2 interact. Evidence of interaction between or among variables is obtained by showing that observations fail to fit well the predictions of a model which assumes no such interactions.

Log-linear models assume that the relative frequencies in the joint distribution should be the product of relative frequencies fixed by margins of the table. The margins which are included in a model specify the interactions among variables which the model assumes to operate, but not a direction of causation (Goodman, 1972).

Every model with a reasonable chance of fitting any of the sets of data in this paper includes the margin [2, 3], which describes the joint distribution of family size and birth order. The reason is that, because of the way birth orders have been defined, the distribution of children in a family among the birth orders (variable 3) is a function of family size (variable 2). Families of size n have $n - 2$ children of middle rank.

By way of illustration, the model [1, 2] [2, 3] assumes interaction between survival and family size, on the one hand, and between family size and birth order, on the order, but independence (multiplicative interaction) between this pair of joint distributions. The model may also be described as assuming the conditional independence of survival (variable 1) and birth order (variable 3), given family size (variable 2). If variable 1 is generalized from "living or dead" to "affected with a disease or not affected," this model is the null hypothesis of the method of Greenwood and Yule (1914).

Technical details are discussed in the Appendix.

4. Results of Computations

The data from Crulai 1 month, Crulai 1 year, Tourouvre-au-Perche 1 month, Tourouvre-au-Perche 1 year, and Geneva will be reviewed in the order just listed.

Below the models in Table 2 are comparisons of models. For example, the comparison 1 − 2 (read "model 1 minus model 2") shows that, in return for giving up 14 degrees of freedom, G^2 is reduced by 13.115, a nonsignificant reduction. Nothing is gained by assuming that survival interacts with family size in the data from Crulai on survival to one month.

Forward selection (see Appendix) leads from model 1 to the model with only one additional margin which makes the greatest improvement in goodness of fit (model 3, which assumes interaction between survival and birth order). Further addition of the remaining possible margin (model 4) does not improve the fit significantly.

Goodman's (1971) backward elimination and Fienberg's (1970) procedure (see Appendix) start from model 4 and eliminate that margin which causes the least (and a nonsignificant) worsening of goodness of fit. The result of deleting margin [1, 2] from model 4 is model 3. Further elimination of either margin leads to a significant worsening of goodness of fit (at the five percent level). Hence backward elimination also rests at model 3.

A probability has been assigned to each model or comparison of models. Although this probability is intended to represent the chance that a value of G^2 greater than that observed would occur by sampling fluctuation alone, the probability assigned may be larger than the

TABLE 2.—Fits and Comparisons of Log-Linear Models: Crulai 1 Month and Crulai 1 Year

Model or Comparison	$\|G^2\|$	df	z	P
Models: 1 month				
1 [1][2,3]	49.604	35	1.65	P = .05
2 [1,2][2,3]	36.489	21		0.01<P<.025
3 [1,3][2,3]	37.905	33	0.64	P = .26
4 [1,2][1,3][2,3]	22.398	19		.25<P<.50
Comparison				
1-2 Survival x family size	13.115	14		.25<P<.50
1-3 Survival x birth order	11.699	2		P < .005
2-4 Survival x birth order	14.091	2		P < .005
3-4 Survival x family size	15.507	14		.25<P<.50
Models: 1 year				
5 [1][2,3]	39.658	35	0.60	P = .27
6 [1,2][2,3]	29.022	23		.10<P<.25
7 [1,3][2,3]	32.283	33	-0.27	P = .61
8 [1,2][1,3][2,3]	17.632	21		.50<P<.75
Comparisons				
5-6 Survival x family size	10.636	12		.50<P<.75
5-7 Survival x birth order	7.375	2		.025<P<.05
6-8 Survival x birth order	11.390	2		P < .005
7-8 Survival x family size	14.651	12		.25<P<.50

Note: In this and all later tables where the following symbols appear,
G^2 = log likelihood ratio;
z = standardized normal transformation of G; and
P = nominal probability that a worse fit between the model and data would have occurred by chance (using a one-tailed test) assuming the model were true.
Source: The data used in the calculation of this table were obtained from Gautier and Henry, 1958.

true probability because of a positive correlation in mortality among members of a sibship, such as Adlakha (1970, p. 87) demonstrated in modern Turkey (see Appendix).

If in Table 2 a nominal one percent significance level had been chosen because simplicity of the model were considered more important, then model 1 might be preferred.

Table 3 shows survival according to birth order along with the maximum likelihood estimates of ${}_{1/12}q_0$, the probability that a newborn child will be dead within one month, by birth order. The firstborn child has nearly twice the probability of dying of the middle or last born.

The data from Crulai on survival to one year are qualitatively very similar. All models considered in Table 2 fit the data acceptably at the five percent level. Model 7, which allows for interaction between birth order and survival, is the choice of forward selection and backward elimination.

The margin which describes birth order and survival appears in Table 3, along with ${}_1q_0$, the probability that a newborn infant will be dead within one year, by birth order. Firstborn children had one chance in four of dying while middle and last born suffered only 70 percent of that mortality. The pattern of differences in mortality by birth order is maintained from one month to one year in this group of children, but the size of the differences decreased absolutely and relatively.

Since, in the data from Crulai, the margin [2, 3] is obligatory for all models, the treatment of survival as a response variable is identical to the analysis just completed. With the data from Tourouvre-au-Perche (four variables), the treatment of survival as a response variable is different.

For the data on the survival to one month of infants in Tourouvre-au-Perche (Table 4), the minimal model, model 1 ([1][2, 3][4]), fits very poorly. Models 2 through 6 add, one at a time, each remaining two-way margin other than [2, 3] to model 1. Only model 6 fits the data at all acceptably, and it fits quite well. In model 6, the distribution of family sizes depends on whether the families were complete (as defined above) or were not complete. Charbonneau (1970, pp. 409–413) gives the number of families by size and type.

The comparisons of models 7, 8, 9, and 13 with model 6 show that the addition of any two-way interaction to those two considered in model 6 never results in an improvement in goodness of fit which is significant at the one percent level. Only allowing for the interaction between survival and birth order (model 7) results in an improvement which ap-

TABLE 3.—Numbers of Children Living or Dead by the Age Indicated According to Birth Order for All Five Sets of Data (Margin [1, 3])

	Birth Order		
Data Set	First	Middle	Last
Crulai 1 month			
Living	230	648	217
Dead	57	90	26
$_{1/12}q_0$	0.199	0.122	0.107
Crulai 1 year			
Living	215	606	201
Dead	72	132	42
$_1q_0$	0.251	0.179	0.173
Tourouvre 1 month			
Living	279	1,399	306
Dead	72	250	45
$_{1/12}q_0$	0.205	0.152	0.128
Tourouvre 1 year			
Living	254	1,196	277
Dead	97	453	74
$_1q_0$	0.276	0.275	0.211
Geneva 20 years			
Living	391	1,099	394
Dead	170	677	167
$_{20}q_0$	0.303	0.381	0.298

$_xq_y$ = (number of children dead at age x + y years)/(number of children living at age y years).

Sources: Gautier and Henry, 1958; Charbonneau, 1970; Henry, 1956.

TABLE 4.—Fits and Comparisons of Log-Linear Models: Tourouvre-au-Perche 1 Month

Model or Comparison		$\|G^2\|$	df	z	P
Models					
1	[1][2,3][4]	523.729	133	16.09	P = 0
2	[1,2][2,3][4]	504.516	114	16.70	P = 0
3	[1,4][2,3]	522.772	132	16.12	P = 0
4	[1][2,3][3,4]	509.755	131	15.77	P = 0
5	[1,3][2,3][4]	515.355	131	15.95	P = 0
6	[1][2,3][2,4]	94.249	89	.43	P = .33
7	[1,3][2,3][2,4]	85.875	87	-.05	P = .52
8	[1][2,3][2,4][3,4]	94.250	87	.58	P = .28
9	[1,2][2,3][2,4]	75.034	73	.21	P = .42
10	[1][2,3,4]	94.250	77	1.36	P = .09
11	[1,2][2,3,4]	75.035	61	1.25	P = .10
12	[1,3][2,3,4]	85.876	75	.90	P = .18
13	[1,4][2,3,4]	93.285	76	1.37	P = .09
14	[1,2][1,3][2,3,4]	66.589	59	.72	P = .24
15	[1,2][1,4][2,3,4]	73.730	60	1.23	P = .11
16	[1,3][1,4][2,3,4]	85.038	74	.92	P = .18
17	[1,2][1,3][1,4][2,3,4]	65.280	58	.70	P = .24
Comparisons					
1-6	Size x type	429.480	44	19.98	P = 0
5-7	Size x type	429.480	44	19.98	P = 0
6-7	Survival x order	8.374	2		.01<P<.025
6-8	Order x type	0.000	2		P = 1
6-9	Survival x size	19.215	16		.25<P<.50
6-13	Survival x type, order x type, and size x order x type	0.964	13		P = 1
10-11	Survival x size	19.215	16		.25<P<.50
12-14	Survival x size	19.287	16		
13-15	Survival x size	19.555	16		
16-17	Survival x size	19.758	16		
10-12	Survival x order	8.374	2		.01<P<.025
11-14	Survival x order	8.446	2		
13-16	Survival x order	8.247	2		
15-17	Survival x order	8.450	2		
10-13	Survival x type	0.965	1		.25<P<.50
11-15	Survival x type	1.305	1		
12-16	Survival x type	0.838	1		
14-17	Survival x type	1.309	1		

Note: For definition of symbols see note to Table 2.
Source: The data used in the calculation of this table were obtained from Charbonneau, 1970.

proaches that level of significance (*P* is between 0.01 and 0.025).

Hence for Tourouvre 1 month, the assumption that the distribution of family size and the type of family interact suffices. The data provide some indication, though not overwhelming evidence, that, as in Crulai, firstborn infants suffer higher mortality than middle or last born; last born suffer very slightly lower mortality than middle born.

When fit to the Tourouvre 1 year data, none of the 17 models in Table 4 is acceptable at the 0.5 percent level. Model 1 in Table 5 shows that all possible two-way interactions do not suffice to describe the data.

Models 2 through 5 in Table 5 add, one at a time, each three-way interaction to the model of all two-way interactions. Only model 3 fits the data acceptably at the one percent level, and this model fits well. Model 3 assumes a three-way interaction among survival, family size, and type of family. One interpretation of this interaction might be that survival depends on family size, but the nature of the dependence is affected by whether or not the family is complete.

Table 6 presents this margin [1, 2, 4] and infant mortality ${}_1q_0$ by family size and type of family. Although overall infant mortality in "other" families is only slightly greater than infant mortality in complete families, there is a greater increase in infant mortality with family size among the complete families than among the others.

Of the remaining two-way interactions in model 3, the interaction between birth order and type of family contributes negligibly (cf. models 6 and 3). Eliminating the interaction [1, 3] between survival and birth order (see Table 3) has an effect which is not significant at the five percent level. The pattern of infant mortality differs strikingly from the pattern so far: infant mortality among first and middle born is high and virtually identical; mortality among last born is about three-quarters of that among first and middle born.

Thus the Tourouvre 1 year data may be described by assuming (model 7) that survival, family size, and type of family interact, and that is all (other than the usual interaction [2, 3] between family size and birth order). The evidence here for interaction between survival and birth order is weak.

Models 5 and 8 to 14 in Table 5 treat

TABLE 5.—Fits and Comparisons of Log-Linear Models: Tourouvre-au-Perche 1 Year

Model or Comparison		$\lvert G^2 \rvert$	df	z	P
Models					
1	[1,2][1,3][1,4][2,3][2,4][3,4]	102.656	70	2.54	P = 0.006
2	[1,2,3][1,4][2,4][3,4]	69.228	42	2.66	P = 0.004
3	[1,2,4][1,3][2,3][3,4]	71.727	66	0.53	P = 0.30
4	[1,3,4][1,2][2,3][2,4]	100.114	68	2.53	P = 0.006
5	[2,3,4][1,2][1,3][1,4]	102.470	60	3.41	P < 0.001
6	[1,2,4][1,3][2,3]	71.736	68	0.36	P = 0.36
7	[1,2,4][2,3]	77.259	70	0.64	P = 0.26
8	[2,3,4][1,2,3][1,4]	69.196	30	4.08	P < 0.0001
9	[2,3,4][1,2,4][1,3]	71.660	56	1.44	P = 0.07
10	[2,3,4][1,3,4][1,2]	100.103	58	3.43	P < 0.001
11	[2,3,4][1,2,3][1,2,4]	38.150	26	1.59	P = 0.06
12	[2,3,4][1,2,3][1,3,4]	64.906	28	3.98	P < 0.0001
13	[2,3,4][1,2,4][1,3,4]	69.777	54	1.47	P = 0.07
14	all 3-way margins	33.918	24	1.38	P = 0.08
Comparisons					
6-3	Order x type	0.009	2		P > .995
7-6	Survival x order	5.523	2		.05<P<.10
5-8	Survival x size x order	33.274	30		.10<P<.50
9-11	Survival x size x order	33.510	30		
10-12	Survival x size x order	35.197	30		
13-14	Survival x size x order	35.859	30		
5-9	Survival x size x type	30.810	4		P < .005
8-11	Survival x size x type	31.046	4		
10-13	Survival x size x type	30.326	4		
12-14	Survival x size x type	30.988	4		
5-10	Survival x order x type	2.367	2		.10<P<.50
8-12	Survival x order x type	4.290	2		
9-13	Survival x order x type	1.883	2		
11-14	Survival x order x type	4.232	2		

Note: For definition of symbols see note to Table 2.
Source: The data used in the calculation of this table were obtained from Charbonneau, 1970.

TABLE 6.—Number of Children Living or Dead at Age One Year in the Tourouvre-au-Perche Sample by Size and Type of Family (Margin [1, 2, 4])

Size of Family	Type of Family					
	Complete			Other		
	Living	Dead	$_1q_0$	Living	Dead	$_1q_0$
4	96	20	0.172	120	64	0.348
5	99	36	0.267	209	51	0.196
6	108	36	0.250	106	38	0.264
7	109	24	0.180	97	36	0.271
8	116	36	0.237	90	38	0.297
9	87	39	0.310	46	26	0.361
10	136	44	0.244	27	13	0.325
11	79	31	0.282	22	0	0.000
12	59	25	0.298	20	16	0.444
13	21	5	0.192	0	0	--
14	20	8	0.286	0	0	--
15	0	0	--	8	7	0.467
16	26	22	0.458	0	0	--
17	15	2	0.118	0	0	--
18	11	7	0.389	0	0	--
Total	982	335	0.254	745	289	0.279

Source: Charbonneau, 1970.

survival as a response variable. Each model includes the margin [2, 3, 4]. Model 5 demonstrates that allowing survival to interact pairwise with each of the remaining variables is inadequate to describe the data. The remaining models add three-way interactions one at a time, two at a time, and three at a time. As in the preceding analysis of these data, all and only those models which include the margin [1, 2, 4] describe the data acceptably. The comparisons of models at the bottom of Table 5 confirm that only the joint interaction of survival, sibship size, and type of family makes a significant difference in the fit of models, regardless of what background interactions are assumed.

The four sets of data (two from Crulai, two from Tourouvre-au-Perche) have yielded identical substantive inferences from analysis which treated all variables as response variables and from analysis which treated only survival as a response variable. The following analysis of the Genevan data treats all variables as response variables. A partial analysis, not reported here, has indicated that treatment of survival as a response variable again gives very similar results. I judged it unnecessary to repeat the full analysis.

Table 7 presents 18 of the models fitted to the data from Geneva. The same kind of analysis described for Crulai and Tourouvre applies here, and a detailed verbal commentary will be replaced by a brief summary.

Models 1, 2, and 3 demonstrate that at least one three-way interaction must be assumed to describe the data and that the margin [2, 4, 5] is necessary and sufficient. That margin, the joint distribution of family size and type by epoch, is given explicitly by Henry (1956, pp. 216–217) and is suggested by his finding that both fertility and mortality among Genevan adults (aged 20 and over) declined markedly between 1550 and 1899 but that fertility and adult mortality changed at different times and rates. The decline in adult mortality shifted the distribution of type of family from incomplete (interrupted

TABLE 7.—Fits and Comparisons of Log-Linear Models: Geneva 20 Years

Model or Comparison	$\lvert G^2 \rvert$	df	z	P
Models				
1 All 2-way interactions	1127.723	872	5.74	P ≐ 0
2 All 3-way except [2,4,5]	987.417	<658	>8.18	P ≐ 0
3 [1][2,3][2,4,5]	686.688	649	1.05	P = 0.14
4 [1,2][2,3][2,4,5]	584.726	639	-1.54	P = 0.95
5 [1,3][2,3][2,4,5]	666.698	647	0.56	P = 0.29
6 [1,4][2,3][2,4,5]	654.447	647	0.22	P = 0.41
7 [1,5][2,3][2,4,5]	455.814	643	-5.65	P ≐ 1
8 [1,2][1,3][2,3][2,4,5]	584.981	637	-1.47	P = 0.93
9 [1,2][1,4][2,3][2,4,5]	540.544	637	-2.80	P = 0.997
10 [1,2][1,5][2,3][2,4,5]	430.892	633	-6.21	P ≐ 1
11 [1,3][1,4][2,3][2,4,5]	631.856	645	-0.35	P = 0.64
12 [1,3][1,5][2,3][2,4,5]	451.477	641	-5.74	P ≐ 1
13 [1,4][1,5][2,3][2,4,5]	443.434	641	-6.01	P ≐ 1
14 [1,2][1,3][1,4][2,3][2,4,5]	539.416	635	-2.78	P = 0.997
15 [1,2][1,3][1,5][2,3][2,4,5]	429.984	631	-6.19	P ≐ 1
16 [1,2][1,4][1,5][2,3][2,4,5]	413.865	631	-6.74	P ≐ 1
17 [1,3][1,4][1,5][2,3][2,4,5]	437.703	639	-6.15	P ≐ 1
18 [1,2][1,3][1,4][1,5][2,3][2,4,5]	412.790	629	-6.72	P ≐ 1
Comparisons for forward selection				
3-4 Survival x size	101.962	10		P < .005
3-5 Survival x order	19.990	2		P < .005
3-6 Survival x type	32.241	2		P < .005
3-7 Survival x epoch	230.874	6		P ≐ 0
7-10 Survival x size	24.922	10		.005<P<.01
7-12 Survival x order	4.337	2		.10<P<.25
7-13 Survival x type	12.380	2		P < .005
13-16 Survival x size	29.569	10		P < .005
13-17 Survival x order	5.731	2		.05<P<.10
16-18 Survival x order	1.075	2		.25<P<.50
Comparisons for backward elimination				
17-18 Survival x size	24.913	10		.005<P<.01
16-18 Survival x order	1.077	2		.20<P<.30
15-18 Survival x type	17.194	2		P = 0.0002
14-18 Survival x epoch	126.626	6		P ≐ 0
13-16 Survival x size	29.569	10		P = 0.001
10-16 Survival x type	17.027	2		P = 0.0002
9-16 Survival x epoch	126.679	6		P ≐ 0
7-13 Survival x type	12.380	2		P < .005
6-13 Survival x epoch	211.013	6		P ≐ 0

Note: For definition of symbols see note to Table 2.
Source: The data used in the calculation of this table were obtained from Henry, 1956.

before the wife reached age 45 by death of one of the spouses) to complete families. The decline in fertility shifted the distribution of family sizes from larger to smaller sizes. Hence a likely candidate for a three-way interaction is [2, 4, 5].

Forward selection among the models in Table 7 thus leads from model 3 to 7 to 13 to 16 to 18. Forward selection identifies the variables which interact with survival in order of importance (most to least) as epoch (see the rightmost column of Table 9), family size (Table 8), and birth order (Table 9). Backward elimination leads from model 18 to 16 to 13 to 7 to 3. Both methods coincide in ranking the variables other than sur-

TABLE 8.—Number of Children Living or Dead by Age 20 in Genevan Sample According to Family Size (Margin [1, 2])

Family Size	Living	Dead	Proportion Dead $_{20}q_0$
2	137	57	0.29
3	248	73	0.23
4	283	101	0.26
5	250	90	0.26
6	169	89	0.34
7	164	102	0.38
8	162	102	0.39
9	130	86	0.40
10	118	102	0.46
11	71	72	0.50
≥12	152	140	0.48

Source: Henry, 1956.

vival by the importance of their interaction with survival. Birth order has no important interaction with survival; the epoch has an extremely important effect on survival.

To describe these data from Geneva requires the same choice between simplicity of model and goodness of fit as have the previous data. All of the models in Table 7 from model 3 onward fit the data acceptably at the ten percent level. Many of the more complex models provide fits which are improbably close. It is only the contrasts between models which justify assuming more than does model 3. The interaction between survival and birth order is significant at the one percent level only in the comparison of model 5 with model 3; both of these models exclude the more significant interactions of survival with family size, type, and epoch.

When Table 9 is compared with Table 3, the pattern of higher mortality in the firstborn in Crulai and Tourouvre-au-Perche is not apparent in the estimates of mortality by age 20, $_{20}q_0$, for Geneva, even in those epochs contemporaneous with the data from Crulai and Tourouvre-au-Perche. Whether this is a real geographical difference, or whether differences in survival by birth order up to one year even out by age 20 is unknown.

The same analysis performed in Table 7, models 3 to 18, for all families from Geneva, is repeated in models 1 to 8 of Table 10 for the complete families from Geneva only. Since all families are of a single type (complete), variable 4 is independent of all others. The three-way margin [2, 4, 5] collapses to [2, 5].

Among the complete families of Geneva, as among all the families, the epoch interacts most importantly with survival, the family size next most importantly, and birth order not significantly (each interaction being considered conditional on the preceding). By contrast with all the Genevan families, among the complete families the change in survival with epoch must be considered in order to describe the data acceptably.

5. SUBSTANTIVE INFERENCES

The substantive inferences from the last section are reviewed here without reference to the log-linear models on which they are based and are compared with the inferences drawn by the original authors.

Each conclusion below reflects a choice between simplicity of model and goodness of fit to the data, a choice which remains subjective to the extent that present statistical theory offers no firm grounds for such choices. The advantage of this form of subjectivity over some less formal treatments of the data is that the levels of probability among which I am exercising choice are explicit in the preceding section.

Crulai 1 month. There is strong, though not overwhelming, evidence that firstborn infants have a chance of dying by one month which is almost twice that of later-born infants, regardless of family size.

Crulai 1 year. Among birth orders, the absolute and relative differences in the probability of dying by one year diminish, though the same pattern of excess mortality among the firstborn remains. The data can be described by assuming that survival to one year is independent of both birth order and family size.

Tourouvre-au-Perche 1 month. As in Crulai, there is strong, but not overwhelming, evidence that firstborn infants have a chance of dying by one month which is greater than that of middle or last born. The data can be described by assuming that family size and the type

TABLE 9.—Number of Children Living or Dead by Age 20 in Genevan Sample According to Birth Order and the Half-Century in Which the Father Was Born (Margin [1, 3, 5]) (Last Column is Margin [1, 5])

Epoch Father Born	Birth Order: First	Middle	Last	All Birth Orders
Before 1600				
Living	55	236	54	345
Dead	53	206	54	313
$_{20}q_0$	0.490	0.466	0.500	0.48
1600-1649				
Living	69	297	61	427
Dead	38	238	46	322
$_{20}q_0$	0.355	0.445	0.430	0.43
1650-1699				
Living	69	220	74	363
Dead	40	124	35	199
$_{20}q_0$	0.367	0.360	0.321	0.35
1700-1749				
Living	55	102	60	217
Dead	21	64	16	101
$_{20}q_0$	0.276	0.386	0.211	0.32
1750-1799				
Living	50	82	51	183
Dead	10	24	9	43
$_{20}q_0$	0.167	0.226	0.150	0.19
1800-1849				
Living	52	98	48	198
Dead	2	12	6	20
$_{20}q_0$	0.037	0.109	0.111	0.09
1850-1899				
Living	41	64	46	151
Dead	6	9	1	16
$_{20}q_0$	0.128	0.123	0.021	0.10

Source: Henry, 1956.

TABLE 10.—Fits and Comparisons of Log-Linear Models: Geneva Complete Families Only

Model or Comparison	$\|G^2\|$	df	z	P
Models				
1 [1][2,3][2,5][4]	335.990	259	3.18	P < .001
2 [1,2][2,3][2,5][4]	239.916	249	-0.39	P = .65
3 [1,3][2,3][2,5][4]	314.642	257	2.44	P = .993
4 [1,5][2,3][2,5][4]	187.706	253	-3.10	P > .999
5 [1,2][1,3][2,3][2,5][4]	239.265	247	-0.33	P = .63
6 [1,2][1,5][2,3][2,5][4]	167.401	243	-3.73	P ≐ 1
7 [1,3][1,5][2,3][2,5][4]	181.713	251	-3.32	P ≐ 1
8 [1,2][1,3][1,5][2,3][2,5][4]	165.432	241	-3.74	P ≐ 1
Comparisons				
1-2 Survival x size	96.074	10		P <.005
1-3 Survival x order	21.348	2		P <.005
1-4 Survival x epoch	148.284	6		P ≐ 0
4-6 Survival x size	20.305	10		.025<P<.05
4-7 Survival x order	5.993	2		P = .05
6-8 Survival x order	1.969	2		.25<P<.50

Note: For definition of symbols, see note to Table 2.
Source: The data used in the calculation of this table were obtained from Henry, 1956.

of family interact but that the survival of a child is independent of family size, birth order, or type of family.

Tourouvre-au-Perche 1 year. The joint interactions among survival, family size, and type of family take the form of a mortality which increases steeply with family size among complete families and of a higher average (but less rapidly increasing) mortality among the remaining families. Conditional upon this interacton, there is no apparent interaction between birth order and survival to one year.

Geneva 20 years. For the ensemble of all families from Geneva, survival to age 20 appears to be independent of family size, birth order, type of family, and epoch. Only an interaction among family size, type of family, and epoch is required to describe the data globally: families get smaller and the proportion of complete families increases. But the data provide very strong evidence for the importance, in decreasing order, of the interactions between survival and epoch, survival and type of family, and survival and family size. Conditional on these interactions, there is no pattern of interaction between survival and birth order.

Separate analysis of the complete families demonstrates that the decline of mortality with epoch must be recognized to describe the data. Conditional on the effect of the epoch on survival (as well as the effect of the epoch on family size), there is some, but not very strong, evidence for a decrease in survival with increasing family size. No pattern of mortality by birth order appears.

I now review briefly the analysis of these data by their original authors.

Crulai

Gautier and Henry (1958, pp. 172–173) include only families of three or more children in their calculations of rates of infant mortality ${}_1q_0$ by birth order. For the first and last born they perform the same calculation that I do in Table 3. For children of intermediate birth order, they weight the infant mortality rate for families of each size by the number of families of that size, rather than by the number of children in families of that size, in order to avoid weighting the overall infant mortality rate of children of middle order in favor of the larger families (which have more such children).

My analysis in Table 2 offers no evidence of a significant variation in infant mortality by family size. Hence the estimate in Table 3 of infant mortality for children of middle order, which weighted each child equally, should not differ greatly from that obtained by Gautier and Henry. It does not. For first, middle, and last ${}_1q_0$'s, Gautier and Henry obtain 0.269, 0.170, and 0.179, respectively. They comment: "There is a very appreciable excess mortality of the first born, which cannot be attributed to chance" but report no statistical test (Gautier and Henry, 1958, p. 173).

Gautier and Henry suppose that mortality during the first month is "endogenous," that is, due to difficulties of childbirth, and that mortality during the remainder of the first year is "exogenous," that is, due to the hazards of the environment. They subtract ${}_{1/12}q_0$ from ${}_1q_0$ for each birth order and observe that the remainders vary much less than do the ${}_1q_0$'s. Hence the differences in mortality by birth order are concentrated in the first month. They infer that the differences in infant mortality by birth order are largely attributable to differences in the ease of first and later deliveries.

Tourouvre-au-Perche

Charbonneau (1970, p. 179) calculates the rates of infant mortality ${}_1q_0$ for first, middle and last born to be 0.276, 0.259, and 0.214, respectively. His rates for first and last born agree very closely with those given here in Table 3. Whether his lower value (0.259 instead of 0.275) for children of middle rank is due to his using the technique of Gautier and Henry is unknown, since he does not describe how he calculates his estimate. Charbonneau (p. 179) comments: "In spite of the presence of numerous incomplete families, the mortality of the last born seems smallest. But the difference between first and last born cannot be considered significant." No statistical significance test is reported to support this conclusion.

Charbonneau estimates endogenous mortality as 0.9 ${}_{1/12}q_0$ and exogenous mortality as the difference between infant mortality ${}_1q_0$ and endogenous mortality. He concludes that the difference (which he previously judged to be nonsignificant) between first and last born is due exclusively to differences in endogenous mortality.

Charbonneau (1970, pp. 180–181) notes that the infant mortality of the firstborn declines steadily with the age at marriage of the mother from 0.323 for mothers married before age 20 to 0.203 for mothers married at age 30 or more. He infers that

> the excess endogenous mortality of first born, pointed out above, may depend only on the relative proportion of women married before age 20. One could also, in this situation, explain the apparent decline in endogenous mortality in the births of later orders, which occur when the woman is older.

Since the infant mortality of last-born children in Tourouvre-au-Perche was 0.211, this explanation is conceivably even sufficient.

A clue to the origin of the three-way interaction [1, 2, 4] found in the data on survival to one year, but not in the data on survival to one month, appears in Charbonneau's demonstration (1970, pp. 174–175) of a clear decline in exogenous infant mortality (that is, in mortality occurring after one month) from 1670–1719 to 1720–1769. (Mortality in the first month of life did not change substantially from the earlier period to the later.) This observation suggests that, as in the data from Geneva, the epoch or historical period is a relevant variable over which the Tourouvre data have been collapsed. If environmental causes of death declined during the period of observation, then the survival of adults through marriage would have improved as well as the survival of infants after their first month; hence the proportion of complete families would have increased. Collapsed over time, the simultaneous decline in mortality and increase in proportion of complete families would appear as an association between survival and type of family. Even though age-specific fertility rates in complete families appear not to have changed from 1665–1714 to 1715–1765 (Charbonneau, 1970, p. 115), the magnitude of the association between survival and type of family would clearly change with the size of the family, since a higher proportion of the smaller incomplete families than of the larger incomplete families would shift to being complete (and to having more children). To say that the interaction between survival and type of family depends on family size is simply to say that there is a three-way interaction among all three variables.

Geneva

Although he reports his observations of families of size two, Henry (1956, pp. 159–164) analyzes only families of size three or larger. To estimate the mortality to age 20 of children of middle birth order, he weights the size-specific mortality by the number of families rather than the number of children. The results Henry (1956, p. 161) obtains do not differ grossly from those in Table 9. As might be expected, his estimate of mortality for children of middle order is slightly lower.

Although Henry cautiously refrains from drawing any conclusions about the relations between birth order and survival, he does point out that in each epoch the last born appear to suffer the least mortality. This pattern appears in Table 9 if the last two epochs (1800–1849 and 1850–1899) are combined to avoid small sample fluctuations in estimating ${}_{20}q_0$. Henry does not affirm the statistical significance of this difference but points out that in recent periods parents who regulate the number of their surviving offspring may well replace a child intended as the last born who dies prematurely (p. 163):

> With risks of death independent of rank, one could nevertheless have a proportion surviving that was higher among last born than among the preceding children resulting from the fact that the children who would have been the last if they had lived did not remain so because they died prematurely. This is an effect not of birth order on mortality, but of mortality on birth order.

Similarly, for the interaction between family size and survival, Henry observes that even in families that are not voluntarily controlling the number of their children, the interval between deaths following the premature death of the earlier born is shorter than when the earlier born lives to at least one year. Hence in families where more children die during their first few months, the final size (number of children born) is higher.

The ineluctable conclusion from Henry's discussion is that the directions of causation underlying the associations established here must be investigated with great delicacy. This conclusion argues, persuasively I think, against the exclusive treatment of survival as a response variable.

Arguments that couples should limit the size of their families because laterborn children have an increased chance of dying find no support in the data on neonatal mortality in Crulai and

Tourouvre-au-Perche or in the data on infant mortality in Crulai. The association between eventual family size (not birth order) and infant mortality in Tourouvre may result from pooling families over a period of declining mortality. Birth order again played a negligible role in explaining rates of survival to age 20 in Geneva, but eventual family size was the most important variable after the overall mortality of the epoch. Hence, at least in these families, it is belonging to a family that is going to become large, rather than being later born, which is unhealthy.

The data from Geneva (Table 8) demonstrate that the probability of survival to the approximate age at which reproduction begins declines as family size increases. (Table 8 exaggerates the increase in probability of dying with increasing family size which could have been observed at any one time, because it collapses the simultaneous decline of mortality and family size over time.) But the average number of children surviving to reproductive age per family (which is family size times the probability of survival) still increases with increasing family size. Hence if the distribution of family sizes in this population is viewed as influenced by genetical evolution under natural selection, mechanisms other than or in addition to that proposed by Lack (1948, 1966) must be invoked to explain the abundance, increasing with time, of smaller families.

Few, if any, modern national statistical bureaus publish data with sufficient internal structure to make possible analyses of contemporary or recent populations of the sort performed here for historical populations. Yet if the relations of childhood mortality to family size, birth order, and parental mortality are to be studied in populations in which parents attempt to regulate their completed family size, such observations of the complete demographic histories of families are essential.

Appendix

This paper fits hierarchical log-linear models to incomplete multidimensional contingency tables by the iterative proportional fitting procedure as implemented in a revision of a computer program originally written by Y. M. M. Bishop (see Bishop et al., n.d.; Mantel, 1970; Fienberg, 1972; Goodman, 1972).

The log-likelihood ratio G^2 is the measure of goodness of fit which is minimized by the fitting procedure:

$$G^2 = |2 \ \Sigma \ (\text{observed}) \ln (\text{observed}/\text{expected})|,$$

where the summation extends over all cells in the table and by definition $0 \ln 0 = 0$. The ratio G^2 has the distribution of χ^2. When the degrees of freedom (df) exceed 30, values of G^2 are converted into standardized normal variates by the approximation $z = (2G^2)^{1/2} - (2\,df - 1)^{1/2}$.

Fienberg (1972, pp. 188–190) describes the procedure which assigns degrees of freedom to the value of G^2 calculated for a given model. As indicated in his correction (Fienberg, 1973), this procedure requires first finding the components which would be separable with respect to the model even if there were no logical zeros in the table and then finding the separations created within these components by the location of logical zeros.

A convenient way has been found to carry out the first step. Suppose the model for a table of d dimensions contains m margins $M_1, M_2, \cdots, M_m$, where each M_i is a subset of the set $D = \{1, 2, \cdots, d\}$ of the first d integers. Then the complete d-way table (assuming no logical zeros) is inseparable with respect to the model if and only if

$$U_{i=1}^{m} (D - M_i) = D.$$

The proof is easy: each margin M_i fixes those dimensions whose numbers are contained in M_i. Hence the cells which are associated in the sense of Fienberg are the cells which are adjacent along the other dimensions, namely, those contained in $D - M_i$. If the union of these dimensions not fixed by the margins is all dimensions, then all cells in the complete table are associated with respect to some margin.

Table A-1 illustrates the assignment of degrees of freedom for a model for the Genevan data. The degrees of freedom lost by fitting the overall mean and the margins of each separate variable are calculated just once for each set of data, since these degrees of freedom are lost regardless of the interactions among variables considered in each particular model. (These are the 22 degrees of freedom, i.e., parameters fitted for main effects in Table A-1.) Each margin M_i in the model (displayed in column 1) which had more than two dimensions implicitly contained three two-dimensional margins (displayed in column 2). The potential number of parameters fitted by each of these margins (column 3) is $\Pi\ (c_j - 1)$, where the product is over all dimensions j contained in the margin M_i and c_j is the number of cells into which dimension j is divided, as in Table 1.

However, some of these margins have either sampling or logical zeros. The

TABLE A-1.—Assignment of Degrees of Freedom (df) for the Model [1, 2, 5] [2, 4, 5] [2, 3] [1, 4] for Geneva, as an Illustration of the General Procedure ($d = 5$, $m = 4$)

Margin M_i	Margins in M_i	Potential Parameters $\Pi(c_i - 1)$	Marginal Zeros e	Cell Zeros Implied Z (M_i)
[1,2,5]		60	33	33x9 = 297
	[1,2]	10	0	-
	[1,5]	6	0	-
	[2,5]	60	16	-
[2,4,5]		120	93	93x6 = 558
	[2,4]	20	0	-
	[2,5]	60	16	-
	[4,5]	12	1	-
[2,3]		20	1[a]	1x42 = 42[a]
[1,4]		2	0	0
		370	160	897

Number of cells in complete table $= \prod_{i=1}^{d} c_i = 1{,}386$.

Parameters fitted for main effects $= 1 + \sum_{i=1}^{d} (c_i - 1) = \sum_{i=1}^{d} c_i - d + 1 = 26 - 5 + 1 = 22$.

Cell zeros implied by two margins:
Z ([1,2,5] ∩ [2,4,5]) = 276
Z ([2,4,5] ∩ [2,3]) = 6
Z ([1,2,5] ∩ [2,3]) = 0

Cell zeros implied by three margins = 0.

Parameters fitted = parameters for main effects + parameters for n way margins, n = 2,3. . .
= 22 + 370 - 160 = 232.

Zero cells implied by margins = 897 - 282 = 615.

d.f. = 1,386 - 232 - 615 = 539.

a- Logical zeros: no children of middle rank in families of size 2.
Source: Henry, 1956.

number of parameters actually estimated for each margin is reduced by the number of these zeros. (Caution is necessary because some arrangements of marginal zeros reduce the number of parameters fitted by more than the number of zeros. For example, $n - 1$ zeros in a margin with only n cells leave no parameters to be estimated. Pathologies of this sort did not occur with the data analyzed here.) The number of zeros in each margin (column 4) must be determined by examination of each margin.

Each of these zeros can only have arisen if all the cells which were summed to give that entry in the margin were also zero and in turn implies that the expected frequencies fitted to those cells will be exactly zero; hence these cells should not be counted among the degrees of freedom of G^2. The number of zero cells implied by each zero in the margin of M_i is Πc_j, where the product is over all dimensions j in $D - M_i$. The zero cells implied by margins which are proper subsets of the M_i have already been counted once among the zero cells implied by M_i and hence are not counted again.

In order to avoid double counting of zero cells implied by more than one M_i, the table of data must be scanned to find the number of zero cells implied jointly by each possible combination of 2, 3 or more margins, each of which has at least one marginal zero. The total number of zero cells implied in the table by the zeros in the margins may then be counted by the principle of inclusion and exclusion.

Corresponding to the model [1, 2, 5] [2, 4, 5] [2, 3] [1, 4] analyzed in Table A-1, the model which treats survival as a response variable is [1, 2, 5] [1, 4] [2, 3, 4, 5]. In this model and all others which treat survival as a response variable, the distinction between structural and sampling zeros disappears. There are simply no observations of survival for those vectors of values of the non-response variables which have zero frequency, whatever the reason for the zero frequency.

Note in Table 2 that, even though the margins are the same in both models 2 and 6, model 2 has fewer degrees of freedom than model 6 because there were no deaths by one month among infants in families of size 13, although there was a death among such infants by one year. Hence there is a zero in the [1, 2] margin for Crulai 1 month, but no zero in that margin for Crulai 1 year.

In testing the hypothesis that birth order does not affect survival (conditional upon the other variables being considered), the null model used to calculate the probabilities assigned to the critical values of G^2 assumes that the survival or death of each individual is independent of the survival or death of all others. But the data analyzed here consist of complete sibships. If members of a sibship were perfectly correlated in their survival or death by a given age, then the probability that the part of G^2 measuring differences across birth orders (conditional on all other variables) would exceed any positive value is zero; each entire sibship lives or dies together. If the positive correlation were less than one, and a test of the null hypothesis that birth order has no effect on a mortality gives a value of G^2 that is significant at (say) the five percent level, then the true probability that such a deviation from the null hypothesis would have happened by chance alone is less than five percent. If the test gives a value of G^2 which is not significant at the nominal five percent level, it is possible that a deviation as large as that observed might happen by chance only five percent of the time.

A detailed test for the presence of correlation in mortality among members of a sibship requires for each family the vector reporting which children lived and which died by a given age, according to birth order. Such details are not available from the published tabulations but are present in the original familial reconstructions on which the tabulations are based. If there is no evidence against independence of birth orders, then the methods of analysis used in this paper are justified, and the probability values are accurate. Otherwise, the models for analyzing marginal homogeneity of Bishop et al. (n.d., Chapters 7, 8) may prove helpful.

The choice of a measure G^2 of goodness of fit does not uniquely determine which model will be selected for two reasons.

First, when there are many dimensions in the contingency table, there are too many hierarchical models for the fit of all models to be measured. Feinberg (1970), Goodman (1971, 1973), Mantel and Brown (1973) and others have advocated various selection procedures which reduce the number of models considered. The forward and backward procedures are illustrated in the text.

Second, more than one model may fit the data adequately by the criterion of goodness of fit. When the true structure of the data is not known, some procedure is needed to avoid more complex models than the purpose of the analysis justifies. Fienberg (1970) recommends eliminating margins from a hierarchical sequence of models arranged in order of decreasing complexity and stopping when the increase in G^2 is significant for the increase in degrees of freedom. With the data being analyzed here, this procedure leads sometimes (e.g., models 13 and 16 in Table 7) to the following awkward situation. For two models, M and M', the probability of a worse fit by chance, assuming the models are true, is very close to 1 (both models are *too* good). The more complex model M' differs from the simpler M by one margin M_1: $M' = M \cdot M_1$. But the difference in goodness of fit between M' and M is wildly significant, that is, the probability of the observed $G^2(M) - G^2(M')$ having $df = df$ (M) $- df$ (M') is very close to zero.

In discussing a similar situation which arises with his data, Goodman (1971, p. 47) comments: "The choice between these two models will depend upon the weight given by the researcher to the advantages of the improved fit . . . and the disadvantages of having introduced additional parameters."

Perhaps, rather than seeking a unique model to describe the data, one should seek a rational method of assigning the contribution to goodness of fit which is due to each of the interactions (margins) which compose a complex model. Better theory here will have immediate utility.

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