Mathematical Models of Conception and Birth,

Mindel C. Sheps and Jane A. Menken. Chicago: University of Chicago Press, 1973. xxiv+428 pp. $18.50.

Three major tools of mathematical studies of populations are the cumulative distribution function (cdf) of nonnegative random variables, Fredholm's integral equation of the second kind, and matrix algebra. Demography introduces the cdf via the life table, which is one minus the cdf of the waiting time from birth to death. Fredholm's equation appears as Lotka's renewal equation for population growth. Matrix algebra makes possible projections of the mean and higher moments of multitype populations (Leslie, J.H. Pollard).

Mathematical Models of Conception and Birth synthesizes clearly and comprehensively new successes of these honored tools. It offers models for the cdf of waiting time from marriage (or regular cohabitation) to first conception or birth. The renewal equation appears in the stochastic theory of renewal processes; its kernel is the probability density function of the interval between births and the solution function is the density of births over time since marriage. The matrix algebra of Markov chains describes the flows of probability density among states defined in terms of susceptibility to impregnation or month of insusceptibility following conception or following various outcomes of pregnancy.

As applied mathematics, *Mathematical Models* offers far more than an analysis of these well-known models. The book builds the models from explicit formalizations of the subject matter. The book shows how computer simulation complements the analysis by paper and pencil. It gives systematic procedures for estimating parameters from data. Thus the book leads from science, the study of unformalized phenomena, into applied mathematics, and back again toward science.

As science, *Mathematical Models* and the two decades of work (by P. Vincent, L. Henry, E. Perrin, R. Potter, the authors and others) on which it is based make clear much that would not be clear otherwise. Given certain assumptions the book shows the relative importance of the effectiveness of a contraceptive, the fraction of a population contracepting, the frequency of abortion, and the duration of various insusceptible periods in affecting the equilibrium birth rate of a marriage cohort of women. Its display of the damped oscillatory response of birth rate to a single step-function change in underlying reproductive and behavioral parameters warns effectively against believing that the early results of programs designed to affect the birth rate will be the same as the long-term results. It analyzes the sensitivity of estimates of demographic parameters and distributions to the sampling frame. (For example, it provides means to decide, for a population that is believed to be stationary but may be stable with an intrinsic rate of natural increase r which is not zero, whether the expectation of life at birth is better estimated by the average age at death of those who die at a given time or by the average age at their eventual death of those who are alive at a given time.) The book makes clear that some things cannot be learned from certain data, for example, when parameters are inextricably confounded. (A marriage cohort of women with a decreasing apparent risk of first conception may have either a fecundity uniform over women and decreasing in time or a fecundity uniform over time but heterogeneous over women; the time course of risk of conception cannot distinguish these possibilities. This result is identical to Proschan's analysis of decreasing failure rate in airplanes.)

In addition to its virtues as applied mathematics and as science, the book is pedagogically excellent. Its exposition is unfailingly clear. For a book of such mathematical complexity, it has very few typographical errors. It offers interesting exercises for the first five chapters. All the mathematics required beyond elementary calculus is developed as needed, although previous exposure to probability theory and statistics will definitely help readers who want to move rapidly to the substantive aspects of the book. I used the book as one of several texts in a course on mathematical models of populations, and covered all but Chapter 6 in less than half a semester merely by forcing the graduate and advanced undergraduate students into slave labor. I found it a welcome psychological relief to begin the course not by talking about the waiting time to death (the life table), but the waiting time to conception and birth.

The book has few defects. The repetition of calculations for substantively identical models once in discrete time and once again in continuous time seems awkward. While the book emphasizes how the discrete and continuous results differ, it does not make clear when one formulation is the limit of the other. The difference in the conventions adopted in Chapter 6 (Markov renewal processes) from those in the rest of the book makes that chapter a bump in the road. Only one set of data is analyzed in the book.

This last, most important, shortcoming emphasizes that this book is not the epitaph of a field but its first major statement. The open opportunities for both empirical and mathematical work are enormous. Data are needed to assess the form and amount

of heterogeneity in fecundity among women and over time, as well as to identify the physiological causes and genetic and demographic correlates of this heterogeneity.

I think the most exciting direction for future mathematical modelling in this immediate area is the integration of these models of fertility history and family building (micro-demography) with classical demographic and with physiological models. Chapter 9 makes important new moves in this direction. Ultimately one might derive from mathematical models of the mammalian ovarian cycle (N. Schwartz) values for the daily fecundabilities which are assumed in *Mathematical Models*. One might then combine family building models with explicit, validated models for the age at first marriage (Coale and McNeil) and other models, yet to be achieved, for the duration of marriage, for nonmarital fertility, and for mortality and other required factors, in order to derive the net fertility function and other functions of macro-demography. These could be linked with social and economic models to determine the nutritional and psychological conditions of individuals which set the values of the parameters appearing in the physiological models. Though some simulations have attempted this task, I would like to see how much could be achieved by more transparent analysis.

It is satisfying that this last book-length work of Mindel C. Sheps, who died in 1973 before it was published, should be a record of so much accomplishment at the same time that this first book-length work of her co-author Jane A. Menken should promise so much more.

JOEL E. COHEN
Hacettepe University