

MATHEMATICS AND MYTH

By Joel E. Cohen

The Saturday before the last national presidential election, the *New York Times* devoted 30 column inches of type to the political leanings of the citizens of Laramie County, Wyoming.

According to John Herbers, "Laramie County, which consists largely of the small city of Cheyenne and endless miles of prairie grass, has a reputation as a weather-vane in national politics because since 1896 it has always voted for the winner in Presidential elections.

"Only two thinly populated rural counties are known to share this bellwether status with Laramie County—Palo Alto County, Iowa, and Crook County, Oregon."

By the end of the article, I remained skeptical that there was anything special politically about these three counties. After all, there are some 3,000 counties in the United States. If I assume each county had a chance of 0.7 of voting for the winner in each of 19 presidential elections, and the behavior of each county were independent of its past and of all others at each election, then I find that just before the 1972 election the expected number of counties voting for the winner in each election since 1896 is nearly three.

Bellwethers Not Bellwethers

While I was unconvinced of the special importance of Laramie County, I was

equally persuaded that my model for the voting of counties did not answer the question whether bellwether counties really exist. I asked Frederick Mosteller if he knew whether the question had been investigated systematically. He referred me to Edward R. Tufte of the Woodrow Wilson School of Public and International Affairs at Princeton University, who sent me a lovely manuscript by Tufte and Richard A. Sun.

With the permission of Professor Tufte, I share some of the manuscript's finer findings, both for their intrinsic interest and for the moral they teach.

Tufte and Sun recorded the votes for the two major parties of 2,938 counties in every presidential election from 1916 to 1968. They then did a series of historical experiments. Prior to the 1968 election they found 49 "bellwether" counties which had voted for the winner every time since 1916 at least 13 elections in a row. In the 1968 election, only 27 (or 55.1 per cent) of these putative bellwethers voted for the winner, while two-thirds of all the counties in the country did so. Hence a randomly chosen county would have been a better bet in 1968, than a bellwether chosen at random.

The same historical experiment was repeated for the so-called "bellwethers" going into the 1964 election, and so on back to 1940, when the bellwethers were chosen as those counties that had voted

for the winner in at least the six preceding elections (back to 1916).

Reviewing the results, Tufte and Sun conclude: "Perhaps each time one hears of an area with a spectacular predictive record in the past, a glimmer of hope and curiosity arises suggesting that surely this fine record couldn't be mere chance—there must be *something* going on. Whatever that something might be, it isn't a high degree of prospective accuracy. Sometimes previously accurate districts do better than just any collection of districts, and sometimes they don't."

In three of the eight elections in their historical experiment (1940, 1960, and 1968), a county chosen at random from the country would have been preferable to a "bellwether" county in predicting the outcome of the approaching election.

Swingometrics Not Much Better

With uncommon generosity, Tufte and Sun applied artificial respiration to what I thought was a dead horse, by also investigating, in the same kind of historical experiments, the predictive value of "barometric" and "swingometric" counties. Surprisingly, the horse got up and at least limped.

A "barometric" county is one whose division of the vote, even if it does not choose the winner, is close numerically to the national division of the vote. Tufte and Sun found that counties with a small absolute average difference from the

national division of the vote in past elections tend to be considerably closer to the future division of the vote than do randomly chosen counties. (In the 14 elections from 1916 to 1968, quiet Ohio County, West Virginia, was the country's leading barometer, unknown even to its own Chamber of Commerce.)

A "swingometric" county is one whose shift in the division of the vote from one election to the next closely parallels the shift in the national division of the vote. Based on the same kinds of historical experiments, "swingometric" counties predict the national shift in the vote better than do randomly chosen counties. The country's leading swingometer is LaPorte County, Indiana.

But the prospective error of both "barometric" and "swingometric" counties far exceeds their retrospective error. "While spectacular in their postdictions," write Tufte and Sun, "these counties are not sufficiently barometric or swingometric in their predictions to provide a precise or reliable guide to upcoming elections. Alternative methods of prediction (the grist of the statistical mill) are also to be preferred because their underlying inferential logic is more certain than the unknown mechanisms producing the highly variable barometric and swingometric behavior observed in our data."

What is the moral? Though the paper by Tufte and Sun contains almost no

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abstract calculation that could be recognized as classical mathematics, their clear thought and quantitative analysis of what is meant by a "bellwether" county have slain a favorite election eve ghost. Now the quadrennial visits to indicator counties, chosen for their past choices of winning candidates, can join Roman augury as a means of foretelling affairs of state.

Such visits are not likely to disappear as rapidly as they deserve to, any more than astrology is on the wane. But for those who are persuaded by mere reason and evidence, this study should be an effective riddance.

Some Other Ghosts

There are many other instances where quantitative, implicitly mathematical, analysis of alleged phenomena (ghosts) has shown that the evidence available was no argument for the existence of the phenomenon, and that far simpler explanations of the evidence could be preferred. The relentless destruction of ghosts is an old tradition in the natural sciences. But it is a tradition which is transferred only with difficulty to human affairs. So I will illustrate the power of mathematics to get rid of bad ideas with a few more human examples.

The first is attributed to Joseph Berkson; it may be apocryphal. Someone reported that pregnancy prevents death from

cancer. Specifically, the proportion of women who survived for five years after discovery of or treatment for some kind of cancer was higher among those who were pregnant during the five year period than among those who were not.

Joseph Berkson is credited, in this story, with discovering that most of the difference between the two groups of women was due to women who were pregnant four or more times in the interval. They almost had to survive five years to get pregnant four times!

Second, it has been widely observed for almost two centuries, and is still believed, that the infant mortality rate (fraction of live born who die by age one) is higher for children who were born third in their family than for children who were born second, and in general increases from the second birth order on. Using a method proposed by Greenwood and Yule in 1915 to analyze some French data which displayed this phenomenon, Louis Henry showed in 1968 that within any family of given size, the infant mortality rate was unrelated to birth order (after the first born).

However, the larger the family the lower the survival rate, even though it was the same for all birth orders within the family after the first child. When individuals were sampled at random, or when all the individuals were pooled together, there was an apparent inverse relation

between birth order and survival rate because higher proportions of the later birth orders came from larger families.

This view of the evidence extends interest in studying birth order as a determinant of mortality (and perhaps of many other characteristics) in finding out what determines a couple's overall fertility and the overall mortality of their children.

A final example is the peculiar frequency distribution of first-cousin marriages. There are four kinds of such marriages: the spouses may be children of two brothers, of two sisters, or of a brother and a sister, in which case the husband may be the son of the brother or the sister. In Austria, for example, 33 percent of first-cousin marriages are between children of two sisters, while only 18 percent are between children of two brothers. Other countries show the same pattern of deviation from an equiprobable distribution.

Various explanations have been advanced, including some which are heavily psycho-analytical. John Hajnal showed however, that the sign of the deviation from equiprobability can be explained by assuming that the probability of marriage of two people is a function of the difference in their ages.

With additional assumptions, based directly on observations, about the age distributions of fathers and mothers at the birth of their first-born and later children, he explained the magnitudes of the deviations from equiprobability. Though simple in concept, the calculations are too elaborate to repeat here. No psychoanalytical ghosts had to be invoked.

"It isn't ignorance that causes the greatest harm," said Josh Billings, "it's knowin' so darned many things that aren't so." Mathematics, wisely applied, is the world's finest defense against being duped by city slickers or other priests, including oneself.

Given all that, it must still be added that the simple mathematical models and quantitative analyses which exorcised the ghosts do not rule out the possibility that ghosts exist. Perhaps psychoanalytically deep motivations do influence the proclivity of first cousins to marry. Perhaps birth order per se does have some small effect on infant mortality or other characteristics, at least in some situations. Perhaps hormonal changes which accompany pregnancy do influence some cancers. All that can be said is that evidence other than that presented is required to make these cases.

And since Laramie County, Palo Alto County, and Crook County all voted with the winner once again in 1972, who can rule out that among them one is a true bellwether? Nevertheless, come 1976, I will not bother to read about them.

(Editor's Note: Joel E. Cohen, an associate professor of Biology in the Faculty of Arts and Sciences and lecturer in Population Sciences in the School of Public Health at Harvard University, is Chairman of the Board of Directors of the SIAM Institute for Mathematics and Society. He is a member of the Committee on Conservation of Nonhuman Primates of the National Research Council, a consultant to RAND Corporation, and Associate Editor of *Theoretical Population Biology*. His current interests are mathematical models of populations, social behavior, infectious diseases, and evolution biology. He has a Ph.D. in applied mathematics and a M.P.H. in public health from Harvard University.) ■