STATISTICS BY EXAMPLE
Exploring Data

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Frederick Mosteller, *Chairman*
Harvard University
William H. Kruskal
The University of Chicago
Richard S. Pieters
Phillips Academy, Andover
Gerald R. Rising
State University of New York at Buffalo
Richard F. Link
Artronic Information Systems, Inc.
with the assistance of
Martha Zelinka
Weston High School, Weston, Massachusetts

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Turning the Tables

JOEL E. COHEN

INTRODUCTION

Sometimes something happens which seems so unusual that we wonder what makes it happen that way. The real question is, what makes us think it is unusual? Most people think something is unusual if it is uncommon. The purpose of this example is to try to make you see how we can count possibilities to help determine whether a given event is really common or uncommon, and thus whether we should think of it as ordinary or as so unusual that it demands some special explanation.

The student cafeteria of a university in California has small square tables and rectangular tables. Some students come into the cafeteria between meals to use the tables for casual conversation and joint studying. A psychologist interested in how people occupy space observed the students for a period of months.

He concluded that 50 pairs seated at the small square tables showed a preference for corner rather than opposite seating because 35 pairs sat corner-to-corner while 15 sat across from one another. He inferred a similar preference for corner seating from a subsequent study of eating pairs in a hospital cafeteria; here, of the 41 pairs he observed, 29 sat corner-to-corner while 12 pairs sat across from one another.

After the rectangular tables in the student cafeteria were modified, but the square tables were left

The author is at Harvard University, Cambridge, Massachusetts.

the same, the psychologist observed for several more months.

He distinguished between those pairs who were interacting (conversing, studying together) and those who were "coacting" (occupying the same table but studying separately). Of the 124 pairs he saw seated at the small square tables, 106 were conversing or otherwise interacting while 18 were coacting. Again he asserted that the interacting pairs showed a definite preference for corner seating, because 70 were corner-to-corner while 36 were seated across from one another. However, coacting pairs chose a very different arrangement. Only two pairs sat corner-to-corner and the rest sat opposite one another. These results, the psychologist claimed, supported the previous inference that corner seating was preferred over opposite seating in a variety of conditions where individuals interact. To the psychologist, the observations suggested that corner seating preserves the closeness between individuals and also enables people to avoid eye contact, since they do not sit face to face.

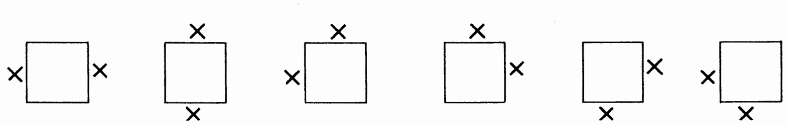
Do the psychologist's data actually show that people prefer corner to opposite seating at small square tables if they are interacting? Do the data support his claim that corner seating is preferred because it enables people to avoid eye contact? What alternative explanations, consistent with his observations, are possible?

For convenience, summarize the data:

	Number of pairs observed	
Interacting pairs	Corner	Opposite
Student cafeteria, first series	35	15
Hospital cafeteria	29	12
Student cafeteria, second series	70	36
Total	134	63
Coacting (not interacting) pairs	2	16

Obviously many more pairs chose corner seating than opposite seating. But before we believe that this shows students prefer corner seating, we need to know what to expect if students show no preferences at all about their seating.

The possible arrangements of two students (×) at a square table are:



Opposite Opposite Corner Corner Corner Corner

There are twice as many corner as opposite arrangements. If students chose at random among these possibilities, then twice as many corner seatings as opposite seatings would be expected.

Another way of seeing the same result is to suppose that the first student of the pair chooses any seat. For the second student, the two seats to the right or the left of the first give corner seatings; only one seat is opposite. If the second student has no preference, then twice as many corner as opposite seatings will occur.

Since the psychologist observed a total of 134 + 63 = 197 interacting pairs, he should have expected approximately $2/3 \times 197 = 131.3$ corner seatings and approximately $1/3 \times 197 = 65.7$ opposite seatings if interacting students showed no seating preferences. The observed data are so close to these expectations that they do not support the conclusion that interacting students prefer corner seating.

Since 18 coacting (not interacting) pairs were observed, 12 corner seatings and 6 opposite seatings would be expected if students showed no preference. Only 2 corner but 16 opposite pairs were observed. Hence it is probably safe to conclude that people who avoid interacting prefer not to sit at a corner with each other.

Even though the proportions in the data agree with those predicted by random seating, we haven't proved the randomness. Some pairs may prefer, or have the habit of, sitting one way, and some another. This could be checked by watching the same couples on several days. Beyond this, it is possible that they sit down at random the first time and then do the same thing thereafter.

What do you think that asking the people in the cafeteria about their preferences would add to the investigation? What new problems of interpretation arise from such interviews?

Exercises

1. Suppose the square tables were placed with one edge against the wall. How many different seating arrangements are possible for a pair of students? How many of these arrangements are opposite and how many corner-to-corner? Does it matter that the psychologist didn't say whether or not the square tables had one edge against the wall?

2. Suppose the cafeteria had circular tables with six chairs each. If a team of four students sat down randomly at one of these tables, what is the chance that the two empty seats would be next to

each other? What is the chance of an arrangement with one student seated between the two empty places? What is the chance that the two empty seats would be opposite each other? From these results, prove that empty chairs usually do not face each other.

3. Tricky question: Assuming four students sit down randomly at a six-seated round table, what is the chance that a student will have empty places on both sides of him? (Remember that in the seating arrangement with one student seated between the two empty places, only one of the four students has empty places on both sides of him. Counting the arrangements from the point of view of an individual student may be different from counting the arrangements of the table as a whole.)