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Men, Models, and Mathematics

Increasingly,

Harvard technologists are using mathematical models to understand such diverse matters as the mating calls of frogs, urban renewal, and the atmosphere of Venus

By Joel E. Cohen

As men have always believed in gods of their own creation, modern men believe in mathematical models of their own making. To an extent that would leave the Pythian priestess of Delphi envious if not incredulous, mathematical models are consulted in questions of poverty and wealth, war and peace, sickness and health, and whom to take out on a Saturday-night date. And the impact of mathematical models is only beginning to be felt.

This year Harvard's Faculty of Arts and Sciences offers courses dealing with mathematical models under at least sixteen different headings. Under another dozen or more course headings the use of mathematical models is so routine that it does not even bear mention.

Though so much instruction—and a great deal of faculty research—is devoted to mathematical models, many people, inside and outside the University, would claim that they do not even know what a mathematical model is. Like Mark Twain, when he learned that he had been speaking prose all his life, such people would be surprised to discover that they have used mathematical models ever since elementary school (even without benefit of "new math"). The discovery is important because insight into simple mathematical modeling can serve as a bridgehead to the understanding of much more complicated modeling.

Such complicated modeling affects our privacy, our incomes, the air we breathe, the food we eat, and our prospects for survival as a species. Less apocalyptically, the beautiful mathematical theories of modern science (theories are comprehensive models with tenure) stand as much a part of world culture as the ethical speculations of major religions, Plato's image of the state, and the music of Bach.

A fruitful model

For an introduction to mathematical modeling, consider the two statements, "One plus one is two," and "One apple plus one apple makes two apples."

The second statement is an empirical statement about the world of experience. If you and I have an agreed-upon procedure for counting, then when we place one apple next to another apple, we agree in counting two apples. The first statement is an arithmetical theorem (though it is not commonly taught as such, nor need it be) which can be proved from simpler axioms, along

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with many other interesting results such as "Two times two is four."

Mathematical modeling is the act of imagination which proposes a connection between the mathematical statement and the empirical observation. In full flower, a mathematical model summarized by "The laws of elementary arithmetic apply to apples and the way we count them" leads to non-obvious predictions like "32,479 apples plus 90,503 apples makes 122,982 apples."

Although this last prediction has probably never been checked empirically, this particular mathematical model is so well confirmed for small numbers of apples that few would doubt it. But the conscientious mathematical modeler cannot rest with proposing, generalizing, and verifying his model in special cases. He must also reach for the limits of validity of his model.

After observing apples in large quantities or making calculations about the strength of materials, the modeler should point out that when more than a hundred thousand apples are gathered together, some of them become applesauce. Ordinary arithmetic does not then apply.

Arithmetic breaks down as a model sometimes even when small numbers are involved. Because of the old grade-school prohibition against adding apples and oranges, no one expects a mixture of a bushel of apples and a bushel of walnuts to yield two full bushels of fruit. But when one quart of clear colorless fluid (water) is added to another quart of the same (rubbing alcohol), some people may be surprised to see that less than two full quarts of clear colorless fluid result. The virtue of having a mathematical model in this case is that the model defines precisely what our expectations are.

The explanation of why the model doesn't work (because the alcohol molecules slip into the spaces between the water molecules, and vice versa) calls for much more sophisticated mathematical modeling which preserves arithmetic on a much smaller physical scale: one molecule plus one molecule equals two molecules.

When the two molecules react to form a single larger molecule, arithmetic breaks down. The mathematics is then reinterpreted so that one electron plus one electron equals two electrons.

Again on the macroscopic level, arithmetic may break down as a model even when identical objects are added together. If one 35-pound lump of uranium is slammed into another 35-pound lump fast enough, the result is not a 70-pound lump but an atomic explosion. Modeling that failure of arithmetic depends on the work of Einstein.

The ultimate reinterpretation of arithmetic so far is that one packet of energy plus another pack of energy makes two packets of energy. Interpreted as the law of the conservation of energy, arithmetic seems to apply without reservation.

These examples should make clear that mathematical modeling does not have one virtue some of its practitioners claim for it. The process of modeling does *not* force the modeler to make explicit all his assumptions about how the real world operates. Even if he says that a particular chunk of the world operates like a particular chunk of mathematics, there is always an incompletely specified gray area in which the abstract objects of mathematics must be interpreted in terms of real operations or vice versa. In the example with the chunks of uranium, the interpretation of "plus" as slamming together may not specify the exact velocity of the collision.

Like the poet, the mathematical modeler assigns names to pieces of experience and tries to construct sentences with those names that yield insight into experience. (The modeler usually has different criteria from the poet for deciding when he has achieved an insight.) The modeler's vocabulary is not limited to arithmetic, nor is his experience to apples and A-bombs.

A swim in the D.E.A.P.

In two evening sessions recently, the faculty of the Division of Engineering and Applied Physics reviewed for students some of their modeling. The Division is home to applied mathematics and to the largest concentration of modelers in the University, but it overlaps and cooperates in the work of many other departments and schools. Here is a sampling of some of the men and their areas of research.

Richard M. Goody, Mallinckrodt Professor of Planetary Physics and Director of the Blue Hill Meteorological Observatory, has been studying the circulation of planetary atmospheres, especially Venus', and the transfer of energy in atmospheres by radiation.

George F. Carrier, Gordon McKay Professor of Mechanical Engineering, has been modeling flows of fluids in oceans and traffic in highways. The combination is no more surprising than the use of arithmetic to count electrons and apples.

In "decision and control," an area which involves, among others, Professors Yu-Chi Ho, Myron B. Fiering '55, and Howard Raiffa (Raiffa holds appointments in business administration and public administration), models have been constructed for the understanding and management of urban renewal and urban housing, optimal trajectories for ascending airplanes, patterns of admissions to Harvard College (to see if personnel could be selected by automatic pattern-recognition techniques), and missile evasion.

The research in decision and control is closely tied to research in the department of economics on planning in developing countries, and to work in the School of Design in city and regional planning.

In the area of environmental engineering, Harold A. Thomas Jr., Gordon McKay Professor of Civil and Sanitary Engineering, has modeled the costs and effectiveness of alternative systems for providing water re-

Karen L. Peterson '72, programming assistant in Adams House G entry, helps out a student mathematician, while the author shares thoughts with the computer about the population dynamics of some Kenyan baboons.

sources in the valleys of the Delaware, Nile, Ganges, and Brahmaputra rivers. He has recently turned to modeling the effects of climatic fluctuations, productivity of land, and rates of work on population levels of the !Kung bushmen in the Kalahari desert in southern Africa. Here he has used data collected by Richard B. Lee in the department of anthropology.

In areas whose relation to engineering and applied physics is even less obvious, William H. Bossert '59, Professor of Applied Mathematics, has modeled the evolution of frogs' mating calls and, with a physiologist at Tufts University, parts of the operation of the human kidney.

In "computational linguistics," Anthony G. Oettinger '51, Professor of Linguistics and Gordon McKay Professor of Applied Mathematics, and Susumo Kuno, Associate Professor of Linguistics, have modeled grammars of languages; William A. Woods, Assistant Professor of Applied Mathematics, has been trying to make mathematically explicit what it means to ask and answer a question.

This fall Richard F. Meyer '54, Professor of Business Administration, teaches a course in the Division on probability models in the social, behavioral, and managerial sciences.

Graduate students are in hot pursuit. To take three random examples, in the Department of Social Relations François P. Lorrain is interpreting concepts of social structure using the mathematical theory of categories. (In that department, one of his professors is Harrison C. White, who holds doctorates in both solid-state physics and sociology and who teaches the departmental course in "Mathematical Models," an always enlightening and generally disorderly plunge into White's current research.) In the department of biology, Madhav Gadgil models the life history of organisms as a problem in the optimal allocation of energy income among needs for maintenance, growth, and reproduction. And applied mathematician Allen Hammond, wearing one hat, has been modeling the behavior of certain ocean waves; wearing another, he has constructed and analyzed models of manpower flows in educational systems.

For undergraduates interested in mathematical modeling, the way is now far easier than it ever has been. The faculty legitimized an undergraduate concentration in applied mathematics just six years ago. For the last few years, Applied Mathematics 115 has been a popular course in "Model Building." In term projects for the course, undergraduates have modeled the effects of the draft law on graduate-school enrolments, the mechanics of a tennis serve, the distribution of commuters among the turnstyles in the rush hour at the Harvard MBTA station and, in possibly similar language, the Battle of Gettysburg.

Systems analysis of a boot locker

A new tool that often makes the new models possible —and at the same time is sometimes confused with them—is the computer. Computers can find the numerical predictions of mathematical models that are simply too complicated to solve using paper and pencil. Computers can also manipulate the masses of data that some mathematical models require, especially those in economics.

And the installation of consoles for Harvard's time-sharing computer in Adams, Leverett, and Kirkland Houses, and in Matthews Hall, has led to a new social form, a three-way computing date: boy, girl, teletype.

What is often not realized by those who view the predictions of "the computer" on election-night television, and even by some of those in the social sciences who use pre-packaged computer programs to analyze laboriously collected data, is that every act of manipulation and analysis (even simple addition, as we have seen) presupposes a model. Just because the model is so complicated it must be buried in a computer is no reason for it to be especially reliable.

Also a newcomer, vaunted "systems analysis" is another, apotropaic name for the frailly human activity of making mathematical models. Usually the chunks of the world modeled in systems analysis are large and complicated. The fundamental laws governing these large chunks may be poorly understood.

Systems analysis is best known for its utility since World War II in assessing the costs and performance of very complicated weapon systems, and as an aid in planning the management of large-scale engineering efforts like space probes and dams and reservoirs for a large river valley. Recently the Congress has called for the application of systems analysis to the solution of pressing domestic problems. New York City has hired the RAND Corporation, a pioneer in the military applications of systems analysis, to help sort out some of New York's internal operations.

Some problems that arise when mathematical modeling moves into domestic life in this way can be illustrated by a simple example.

Suppose an elementary school needs a boot locker, in which the children can store their boots on snowy days when they leave the playground. The problem calls for basic research and engineering design.

The problem in basic research is to find the relation between the number of children and the number of feet on the playground, and between the number of feet and the number of boots. Prior basic research, by ivory-tower students of school children, may have established that the number of feet is twice the number of children, barring amputees, and that the number of boots is never greater than the number of feet; and on very snowy days, boots and feet are equal.

If the answer does not exist already, then basic research must be done to determine the relation between the number of children and the number of boots (or the relation between population density and the crime rate, or the relation between levels of education and rate of growth of the gross national product).

If only aggregated data are available to the basic researcher, he may try (with poor results) to correlate the average grade on the prior spelling test with the number of boots on the playground. But if the researcher is permitted to look at whole children, he may note that each of them has two feet, and each foot wears one boot.

Such observations invade the privacy of the children. The researcher may be tempted and have the opportunity at the same time to ask whether the number of boots the child wears is also related to race, history of bedwetting, and parents' income.

In some areas of research, there is a social cost (like loss of privacy) to knowledge which can be used for social benefits. The systems analysis that can make explicit the cost-benefit accounting of social knowledge does not begin to exist, for no one can evaluate the opportunity cost of our ignorance about society.

Given the needed relation from basic research between maximum number of boots and number of children, the engineering design problem is to determine the cost of boot lockers of varying capacities, the cost of insufficient capacity (how do you measure the cost of upsetting children, or crowding people at a public beach?) under peak loads, and an estimate of the demands to be made on the locker.

If the principal tells the engineer the number of children in the school and insists that the locker just accommodate the boots of every one of them, the engineer must merely solve the equation "Boots equals two times children" and build the locker. This problem is like the nice problems with unique answers in high-school algebra, where the number of equations equals the number of unknowns.

More typically, the engineer is faced with two complications: overdetermination and underdetermination. If the engineer has to estimate the number of children in the school, and his counts of the playground differ from day to day, then he has more than one number to describe the true number of children, which is therefore overdetermined. So he has to use statistical techniques of estimation.

The same problem could have come up earlier if basic research had not provided him the true relation between numbers of boots and feet. Since, in most social applications of mathematical modeling and systems analysis, both the fundamental laws and the magnitudes of the variables involved are not known, mathematical models are often very data-hungry. Data are often expensive, and statistics is a difficult science to use properly.

The engineering problem is underdetermined if the data do not completely determine a solution. Suppose the principal does not require that the locker be able to hold every child's boots. Then even if the engineer can estimate the distribution from day to day of the number of boots that require locker space, he must pick a criterion of performance (an objective function, in economic lingo) to optimize.

This objective function, so called because it characterizes the objective of the plans to be made, is in fact anything but objective: it may reflect the interests of the engineer (maximize profit, or minimize time spent on the project) or of the principal (take up as little space as possible with the locker, but make it big enough so that the parents don't complain) or of the children (make the locker big enough to play in). Or the objective function may be some weighted combination of all these and other interests.

The weights attached to conflicting interests in the objective function for any social project are not given by laws of nature (or if they are, we do not understand nature well enough yet to see how). At present the weights must be chosen. If the society does not make sure that an objective function represents a desirable balance of its interests, then the engineer, subject to what special influences are exerted on him, must choose. The resulting plan (for a boot locker, welfare scheme, or national park system) may very well carry the halo of systems analysis without serving the common interest.

Inventions, some better than others

From this safari through arithmetic as a model for the addition of fruit, some modeling at Harvard, and the systems analysis of a boot locker, some general points ought to be clear.

Mathematical models are inventions. Neither pure mathematics nor pure empirical work, models establish tension between the conceptual structures of the mathematician and the recalcitrant world of experience. Those men and women who are lucky enough to work at building mathematical models constantly discover the rewards of using man's clearest, most public thought, mathematics, to help make sense of the world: a good little guess about a fundamental structure of nature brings big returns in understanding and order. Mathematics is unreasonably useful in suggesting good little guesses.

Like any other acts of imagination, some models are better than others. All have their limits of validity. (Some modelers, especially biologists and economists, appear unconcerned that their models illuminate nothing at all in the real world, but are also trivial mathematics. The results obtained by these modelers suggest the discovery of the dachshund described by Wassily W. Leontief, Henry Lee Professor of Economics. This particular dog, according to Leontief, backed his rear end up against a tree to do business, and when his front end came poking around the tree from the other direction, he exclaimed, *"Zut, c'est moi!"*) Whether he is speaking for scientific or public consumption, the modeler ought to emphasize the limits on the validity of his model, as best he understands them.

People are constructing models, of greater or lesser usefulness, for practically everything. A new generation of modelers is being trained in an activity that hardly was recognized a generation ago.

If the society wants the help of systems analysis (big mathematical models) to solve its social and ecological problems, it must recognize that the models are no better than the systems equations in them. Systems equations arise from basic research. Basic research is costly, both in the opportunity cost of bright men and scarce materials, and, in some cases, in the possible threat to the autonomy and privacy of individuals who may become the subjects, as well as the beneficiaries, of the research.

When it buys social applications of basic research, the public needs to remember that models are implied by the ways data are manipulated; a sophisticated public may want those models made explicit. At the same time, since the solution of most social problems is underdetermined, the public may want to make sure its interests are represented in the engineer's objective function.

The balancing of conflicting interests is a political, not a mathematical, problem. Political work may be necessary to assure that mathematical models for social applications properly represent people's wishes.

Even among scientists considered adepts in the creation, exploration, and scientific application of mathematical models, some of these points have not always been clear. Such confusion assumes public significance as more and more technological professionals apply mathematical models to practical questions.

Without mathematical training, no one can check a mathematical argument to the root. But mathematical models are mathematics peculiarly embedded in the world of experience. With experience and education it is possible to sense the powers and limitations of mathematical models.

Unless responsible men at large, and especially those nonmathematically trained men in positions of making policy, have a sense of these powers and limitations, they will have to accept on faith the divinations of those who claim to interpret the oracles.