



## Further Properties of Third Order Determinants

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### FURTHER PROPERTIES OF THIRD ORDER DETERMINANTS

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C. W. Trigg (MATHEMATICS MAGAZINE 35: 78, 1962) presents the following property of third order determinants and their related "twists:"

$$(1) \quad \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + \begin{vmatrix} a_1 & b_1 & c_1 \\ c_2 & a_2 & b_2 \\ b_3 & c_3 & a_3 \end{vmatrix} + \begin{vmatrix} a_1 & b_1 & c_1 \\ b_2 & c_2 & a_2 \\ c_3 & a_3 & b_3 \end{vmatrix} = 0.$$

A different, but equivalent, representation of this result reveals an underlying symmetry which suggests two further analogous properties.

Let  $V_i = (a_i, b_i, c_i)$ ,  $i = 1, 2, 3$ . Define  $s^0 V_i = V_i$ ,  $s^1 V_i = (c_i, a_i, b_i)$ ,  $s^2 V_i = (b_i, c_i, a_i) \cdots$ . Then (1) takes the form

$$(2) \quad V_1 \cdot (V_2 \times V_3) + V_1 \cdot (s^1 V_2 \times s^2 V_3) + V_1 \cdot (s^2 V_2 \times s^1 V_3) = 0,$$

where " $\cdot$ " means scalar product and " $\times$ " means vector product. The left member of (2) equals  $V_1 \cdot (V_2 \times V_3 + s^1 V_2 \times s^2 V_3 + s^2 V_2 \times s^1 V_3)$ , and the right hand factor of this product must be 0, since (2) holds for all  $V_1$ . For arbitrary  $(V_2, V_3)$ , define

$$[s^0 \times s^0 + s^1 \times s^2 + s^2 \times s^1](V_2, V_3) = V_2 \times V_3 + s^1 V_2 \times s^2 V_3 + s^2 V_2 \times s^1 V_3.$$

Then

$$(3) \quad s^1 \times s^2 + s^0 \times s^0 + s^2 \times s^1 = 0,$$

i.e., the left member annihilates every pair of 3-vectors. The form of (3) suggests that

$$(4) \quad s^0 \times s^2 + s^1 \times s^1 + s^2 \times s^0 = 0$$

and

$$(5) \quad s^0 \times s^1 + s^2 \times s^2 + s^1 \times s^0 = 0,$$

both of which may be easily verified.

These results also hold for vertical permutations. If  $*V$  is the column vector which is the transpose of  $V$ , and  $W$  is a column 3-vector, define  $tW = *s*^{-1}W$ . Then we may substitute  $t$  for  $s$  in (3), (4), and (5), since the value of a determinant is not affected by interchanging rows and columns.