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FERTILITY INCENTIVES AND PARTICIPATION IN LOCALITIES WITH LIMITED MEANS: A DYNAMIC MODEL OF PER CAPITA RESOURCES

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Several countries have attempted to change human fertility through economic incentives. This paper presents simple mathematical models of the participation of couples in a locally funded program of economic incentives. The models take as a springboard China's one-child program. Localities with a low per capita incentives attract few couples to the program, while localities with high incentives attract many couples at first but the value of the benefits is then watered down. The models show that participation in the program may persistently oscillate or may decay to a stationary level. Which behavior occurs is determined by whether there are decreasing, constant, or increasing returns in the rates of participation in response to successive equal increments in the incentive offered, and by the extent to which prospective parents learn from experience with past oscillations in the incentives. The models raise many empirical questions about the dynamics of incentive programs.

KEY WORDS: Family planning, economic incentives, fertility modification, nonlinear dynamics, program participation, mathematical models.
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1. INTRODUCTION: HOW DO LOCAL RESOURCES AFFECT PROGRAM PARTICIPATION?

Several countries have attempted to lower or raise fertility through economic incentives (Ferreira 1986). According to a 1987 questionnaire survey of one hundred developing countries with populations over one million, six Asian countries (Bangladesh, India, Korea, Nepal, Pakistan, and Sri Lanka) pay acceptors of a sterilization program, and two of these (Bangladesh and India) also pay acceptors of an intrauterine device program (Ross and Isaacs 1988). Payments are often presented as compensation for the time and expenses required to participate in the program, but are cash incentives nonetheless. For example, Thapa et al. (1987) and de Silva et al. (1988) describe payments to vasectomy acceptors in urban Sri Lanka. On the other hand, the Province of Québec (Canada), Belgium, France, and several formerly Communist countries of Europe have attempted or are attempting to raise fertility through economic incentives (Ekert-Jaffe 1986; Blanchet 1987).

Governments may use economic incentives to influence individual behavior with significant social externalities for a variety of purposes besides lowering or raising fertility. State-run lotteries may be used to influence savings and investment. The time-series of traffic tickets issued in Taiwan from 1977 to 1989 showed pronounced peaks in 1977, 1980, 1982, 1987 and 1988 (Chu 1990). During these episodes, "sweeping traffic rectification" campaigns tightened enforcement of traffic laws. Subsequently, traffic violations dropped dramatically, enforcement was relaxed, and another cycle began.

This paper presents a simple mathematical model of the participation of individuals, couples or families in a locally funded program of economic incentives. The model takes as its point of departure China's policy of one child per family, launched in 1979, but is not closely tied to the details of that program. For example, the model pays no attention to whether the incentives are intended to raise or lower fertility, and could in principle be applied to some behaviors other than fertility.

Implementation of China's family planning policy of one child per family faced many difficulties, including one described by Bongaarts and Greenhalgh (1985, p. 593): "Administrative decentralization also creates economic problems that compound enforcement difficulties. Under the one-child policy, one of the major motivations for limiting childbearing is the expectation of economic incentives, including wage supplements and priority in housing, schooling, medical care, and the like [I omit the citations to original sources] .... The costs of these incentives are borne by local work units, whose resources vary widely. Where units are poor, few couples sign up for one-child certificates, because benefits are poor or fail to materialize .... Where units are rich, many couples sign up, and the value of the benefits is watered down: everyone having priority is tantamount to no one having priority ...."

The models presented here show when the behavior described by Bongaarts and Greenhalgh (1985) may be expected to occur and to persist. The models make it possible to answer some specific questions, given explicit assumptions. How does the program participation rate differ between localities with different levels of economic incentives, all else being equal? If many couples join a program, and the per capita benefits are diluted, what will happen next? If couples drop out of a program because the expected benefits fail to materialize or are diluted, will the resources made available by the departure of those couples cause a subsequent increase in program participation? If so, will the fluctuations in the participation rate persist or disappear?

Bongaarts and Greenhalgh (1985) suggested that the level of participation depends on the per capita budgets available. The dynamic models proposed here belong to a family of models in which population dynamics are assumed to depend on per capita resources (e.g., Getz 1984; Arditi and Ginzburg 1989).

Section 2 describes the construction of a simple dynamic model to represent the participation of couples in a reversible program (i.e., one which couple can join or leave at will) when the program provides incentives that are locally funded. Section 3 gives a nontechnical review of the model's behavior when the so-called participation function (defined below) is assumed to be always convex, or always concave, or always linear. Section 4 describes the model's behavior when the participation function is allowed to have a point of inflection. Section 5 describes the model's behavior when prospective parents respond to the weighted average incentive offered since the beginning of the program, and not merely to the incentive offered most recently. Sections 4 and 5 are based largely on numerical calculations. Section 6 answers the specific questions raised in this introduction and raises further empirical and theoretical questions. The main exposition is nontechnical and uses only elementary algebra. The mathematical analysis of the model of Section 2 is confined to an appendix.

2. CONSTRUCTION OF A DYNAMIC MODEL FOR PROGRAM PARTICIPATION

Consider a locality with N eligible couples and an annual budget for incentives of B (measured in units of currency, housing, food, or any appropriate single yardstick). This budget can provide an annual incentive (per capita, taking a couple as a unit) of $C = B/N$. Suppose that an incentive of a or less will attract nobody into the program, while an incentive of b or greater will attract everybody into the program. The interesting case to be considered here is that in which the incentive C is enough to attract some people, but not everybody; hence assume $a < C < b$. The per capita budget is assumed constant, i.e., B and N are assumed to change either not at all or proportionally.

For each possible incentive x between a and b, suppose that the fraction of couples who participate in the program is $F(x)$, when x is announced (at the beginning of the first year) or observed (during the year before the second or later years). For each incentive x, $F(x)$ is thus a number between 0 and 1. The function F will be called the participation function. (In classical economics, the supply function specifies the amount of a commodity that will be produced in response to a given market price. The participation function is an analogous concept that gives the participation rate as a function of the incentive.) One way to think about the participation function is to imagine that each couple in the locality has its own reserve price. If the incentive is below the reserve price, the couple will not participate; if the incentive is at or above the reserve price, the couple will participate. From this viewpoint, $F(x)$ is the fraction of couples whose reserve price is at or below the offered incentive x.

Since the incentive announced at the beginning of the program is C, the fraction of couples who will participate in the first year of the program is $p_1 = F(C)$. The actual number of couples will be $NF(C)$. Assume that the locality aims to spend its entire annual budget B on the couples who choose to participate, because it does not like to be seen as having surpluses, or because it cannot stockpile unexpended resources from one year to the next, or for other reasons. Then the actual incentive available during the first year will be the budget divided by the number of participating couples, i.e., $B/(NF(C))$. Since $B/N = C$, the available incentive may also be written $C/F(C) = C/p_1$. Suppose also that, in order to avoid charges of extravagance, the locality never spends more than b on each couple that participates, i.e., if the available incentive $C/F(C)$ exceeds b, then the actual incentive expended during the first year is just b. Now let $x_0 = C$ be the incentive initially announced (prior to the first year of the program) and let x_1 be the incentive delivered during the first program year. The preceding assumptions about the behavior of couples and of the locality are summarized by

$$x_1 = \min\left(b, \frac{C}{p_1}\right) = \min\left(b, \frac{C}{F(C)}\right) = \min\left(b, \frac{C}{F(x_0)}\right). \tag{2.1}$$

Seeing that the actual incentive during the first year was x_1, the fraction of couples who will sign up for the second year of the program is assumed to be $p_2 = F(x_1)$, and hence the actual incentive during the second year of the program will be $x_2 = \min(b, C/p_2) = \min(b, C/F(x_1)$. In general, the incentive in succeeding years will be determined by

$$x_{t+1} = \min\left(b, \frac{C}{p_{t+1}}\right) = \min\left(b, \frac{C}{F(x_t)}\right), \qquad x_0 = C, \qquad t = 0,1,2,\ldots. \tag{2.2}$$

The fraction of couples who participate is likely to be of greater practical interest than the incentive itself. To describe the dynamics of the fraction of couples who participate, (2.2) may be rewritten in an equivalent form that involves participation rates instead of incentives. Since N is finite, the fraction of couples who participate must be some integer multiple of $1/N$. Assume that N is large enough that it is reasonable to treat the fraction of couples who participate as a continuous variable, i.e., as a quantity that can vary smoothly from 0 to 1. Write the fraction of couples who participate in year t of the program as p_t. Thus $p_1 = F(C) = F(x_0)$, $p_2 = F(x_1)$, and generally $p_{t+1} = F(x_t)$. Then taking F of both sides of (2.2) and shifting t by 1 gives

$$p_{t+1} = F\left(\min\left(b, \frac{C}{p_t}\right)\right), \qquad p_1 = F(C), \qquad t = 1,2,3,\ldots. \tag{2.3}$$

This equation depends on the units used to measure incentives, because these units determine the magnitudes of b and C. To eliminate that dependence, henceforth let incentives be measured by the fraction of the distance that they are between a and b; that is, the incentive x in dimensional units is now replaced everywhere by $(x-a)/(b-a)$ in dimensionless units. Thus the incentive b sufficient to attract all couples becomes the incentive 1 in dimensionless units; the incentive a sufficient to attract no couples becomes the incentive 0 is dimensionless units; the incentive C in dimensional units corresponds to a dimensionless incentive $c = (C-a)/(b-a)$; and F is rescaled to dimensionless form so that $F(0) = 0$ and $F(1) = 1$. In dimensionless form, (2.3) becomes

$$p_{t+1} = F\left(\min\left(1, \frac{c}{p_t}\right)\right) \equiv G(p_t), \qquad p_1 = F(c), \qquad 0 < c < 1, \quad t = 1,2,3,\ldots. \tag{2.4}$$

It now remains to specify the participation function F. At a minimum, it is reasonable to expect that more incentives should attract more couples. Hence F will be assumed to be strictly increasing.

How many additional couples will each additional small increment in the incentive attract? Three possibilities will be considered in detail: decreasing returns; constant returns; and increasing returns.

If each small increment of a given size in the incentive attracts a diminishing increment of couples to the program, then the participation function displays diminishing returns, or is concave. Diminishing returns would occur, for example, if the eagerest couples were attracted into the program by even a small incentive, but increasingly larger incentives were required to attract increasingly reluctant couples.

If each small increment of a given size in the incentive attracts the same additional fraction of couples, then the participation function displays constant returns, or is linear.

If each small increment of a given size in the incentive attracts an ever larger increment of additional couples, then the participation function displays increasing returns, or is convex. Increasing returns might occur if there were a bandwagon effect, so that the more couples there were already participating, the easier it became for an additional increment in the incentive to attract additional couples.

Many mathematical functions could represent increasing or decreasing returns. At the moment, there appear to be few or no data to choose among them. Hence, in a first analysis like this one, a mathematical function may be chosen on the basis of its simplicity and convenience. Assume

$$F(x) = x^q, \qquad 0 \le x \le 1, \quad 0 < q < \infty. \tag{2.5}$$

Figure 1 plots the participation functions (2.5) with $q = 1/2$, 1, and 2. These functions illustrate decreasing returns, constant returns, and increasing returns, respectively, and will be the participation functions used except for Section 4.

It seems reasonable to suppose that any concave or convex strictly increasing participation functions F with $F(0) = 0$ and $F(1) = 1$ would share the qualitative properties of (2.5) with $q = 1/2$ and $q = 2$, respectively. The participation function (2.5) has decreasing returns whenever $0 < q < 1$, constant returns when $q = 1$, and increasing returns whenever $1 < q < \infty$. Using the participation function (2.5) in the dynamic equation (2.4) for the participation rate gives an iteration rule that predicts (according to the model) the program participation rate in the following year as a function of the program participation rate in the present year:

$$p_{t+1} = \left[\min\left(1, \frac{c}{p_t}\right)\right]^q \equiv G(p_t), \qquad p_1 = c^q, \qquad 0 < c < 1, \quad 0 < q < \infty,$$
$$t = 1, 2, 3, \ldots . \tag{2.6}$$

Figure 2 graphs the iteration rules G for all combinations of decreasing, constant and increasing returns ($q = 1/2, 1, 2$) and low and high budgets ($c = 1/3, 2/3$). It is evident from Figure 2 that, for any given value of q and any given participation rate this year, the participation rate will be higher next year if the per capita budget c is higher. For any given budget and any given participation rate this year, the participation rate will be higher next year as the participation function changes from increasing, to constant, to decreasing returns. This difference is an obvious consequence of Figure 1, because for any given incentive, the participation rate is higher

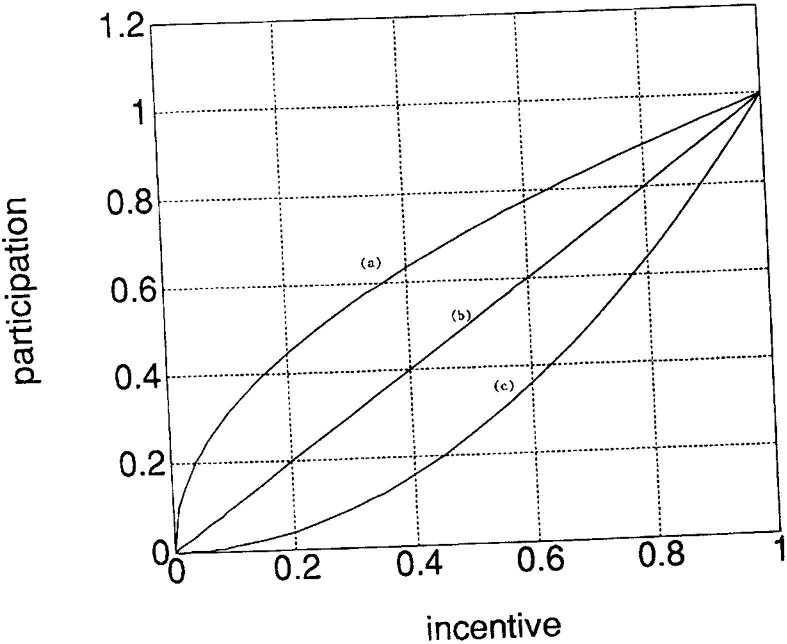

FIGURE 1. Participation functions give the fraction of couples who participate in the program as a function of the incentive. The dimensionless incentive, measured as a fraction of the distance from a completely ineffective incentive to a completely effective incentive, is denoted by x. The fraction who participate when observing incentive x in a previous year is denoted by $F(x)$. (a) Decreasing returns: $F(x) = x^{1/2}$. (b) Constant returns: $F(x) = x$. (c) Increasing returns: $F(x) = x^2$.

with decreasing returns than with constant returns, and higher with constant returns than with increasing returns.

The behavior of (2.6) is analyzed in the Appendix. The following section summarizes the results nonmathematically.

3. BEHAVIOR OF THE MODEL WITH DECREASING, CONSTANT, OR INCREASING RETURNS

The rates of participation of couples in the fertility incentive program will change over time in different ways, depending on whether there are decreasing, constant or increasing returns in the rates of participation in response to successive smal equal increments in the incentive offered.

When the participation function shows decreasing returns, then the participatio rate rapidly converges to a single fixed value, given explicitly by the formula for p^* in (7.2). This behavior holds whatever the initial incentive, so it does not matte whether the locality initially announces its incentive as the budget per capita or an other arbitrary figure; the limiting participation rate is determined solely by the bud get per capita (denoted by c) and the extent of decreasing returns (measured by q). Moreover, if the participation rate is near p^* and then is perturbed away from p^* e.g., by a random fluctuation or an exogenous shock, then (provided the budget pe

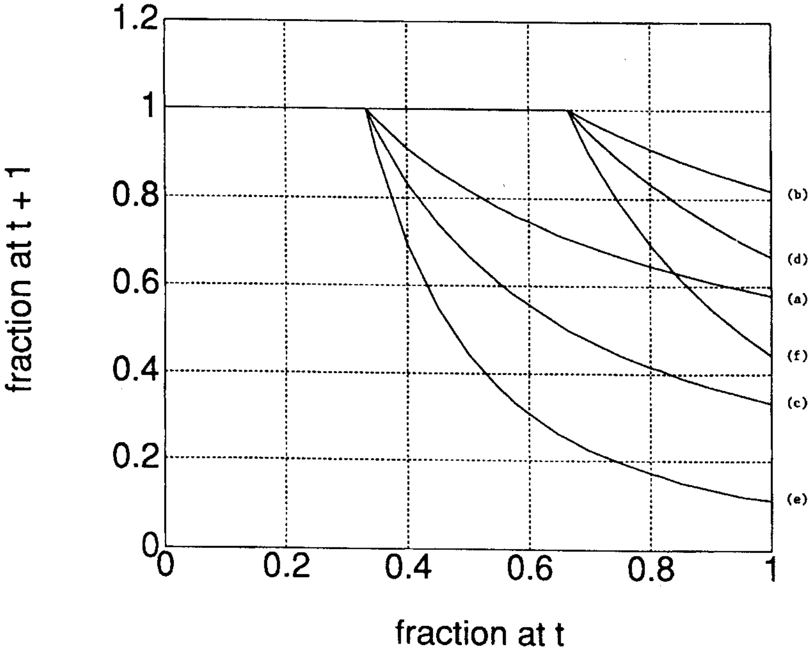

FIGURE 2. Iteration rules predict the program participation rate in the following year as a function of the program participation rate in the present year, according to (2.6) with (a) decreasing returns ($q = 1/2$) and low budget ($c = 1/3$); (b) decreasing returns ($q = 1/2$) and high budget ($c = 2/3$); (c) constant returns ($q = 1$) and low budget ($c = 1/3$); (d) constant returns ($q = 1$) and high budget ($c = 2/3$); (e) increasing returns ($q = 2$) and low budget ($c = 1/3$); and (f) increasing returns ($q = 2$) and high budget ($c = 2/3$).

capita c and the parameter q remain constant) the participation rate will gradually return to p^*. While approaching its limit p^*, the participation rate may overshoot p^*, and then fall below p^*, as shown in Figure 3(a, b). This behavior corresponds to that described by Bongaarts and Greenhalgh (1985): a high participation rate dilutes the incentive actually received, and participation subsequently falls. However, overshoots are temporary. Fluctuations are gradually replaced by an increasingly steady level of participation. The steady level p^* increases with an increasing per capita budget c. Hence the higher the per capita budget c, all else being equal, the higher the long-run average participation (compare Figure 3(b) with Figure 3(a)).

When the participation function shows constant returns, then the participation rate never converges to a single fixed value unless it just happens to start at p^*. Rather, the participation rate fluctuates back and forth between two values. One of these two values equals the initial incentive, so it matters what the locality initially announces as its incentive. The long-run average participation rate, i.e., the average of the two alternating values of the participation rate, is never less than p^*. The higher the locality's per capita budget c, the higher the long-run average of the participation rate (compare Figure 3(d) with Figure 3(c)). If the participation rate is perturbed at any time, then it enters a new two-point cycle starting from wherever it happens to fall; it does not return to its previous cycle.

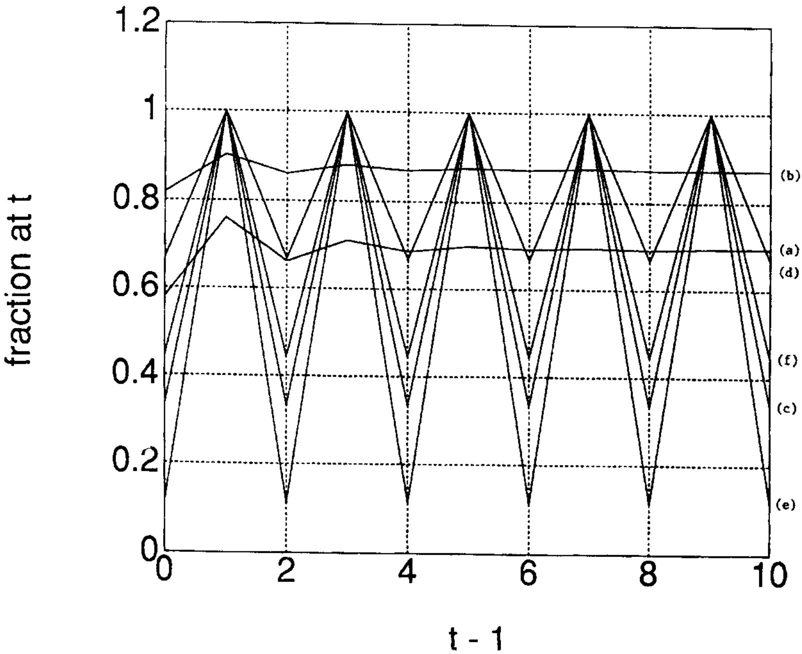

FIGURE 3. Trajectories in time, or forward orbits, of the program participation rate according to (2.6) with (a) decreasing returns ($q = 1/2$) and low budget ($c = 1/3$); (b) decreasing returns ($q = 1/2$) and high budget ($c = 2/3$); (c) constant returns ($q = 1$) and low budget ($c = 1/3$); (d) constant returns ($q = 1$) and high budget ($c = 2/3$); (e) increasing returns ($q = 2$) and low budget ($c = 1/3$); and (f) increasing returns ($q = 2$) and high budget ($c = 2/3$). The exact formulas for these trajectories are given in (7.4) and (7.5).

When the participation function shows increasing returns, the participation rate never converges to a single fixed value unless it just happens to start at the fixed point p^*. If the participation rate does not start at p^*, then it alternates between two values. These values are determined by the budget per capita c and the extent of increasing returns (measured by q), and not at all by the initial incentive. The long-run average participation rate (that is, one half the sum of the high value plus the low value) may be greater or less than p^*. Thus the effect of cycling may be favorable or unfavorable to the average participation rate. The higher the locality's per capita budget c, the higher the average participation rate (compare Figure 3(f) with Figure 3(e)). If the participation rate is perturbed at any time, it returns to the two values of its previous cycle. The return to the stable cycle may be gradual or immediate, depending on the particular perturbation.

In the cases of constant and of increasing returns, an overshoot followed by a fall of participation, as described by Bongaarts and Greenhalgh (1985), is a permanent characteristic of the behavior of couples. Low participation is always followed by an overshoot. The extent of the oscillation is stable against perturbations in the case of increasing returns, but is sensitive to perturbations in the case of constant returns.

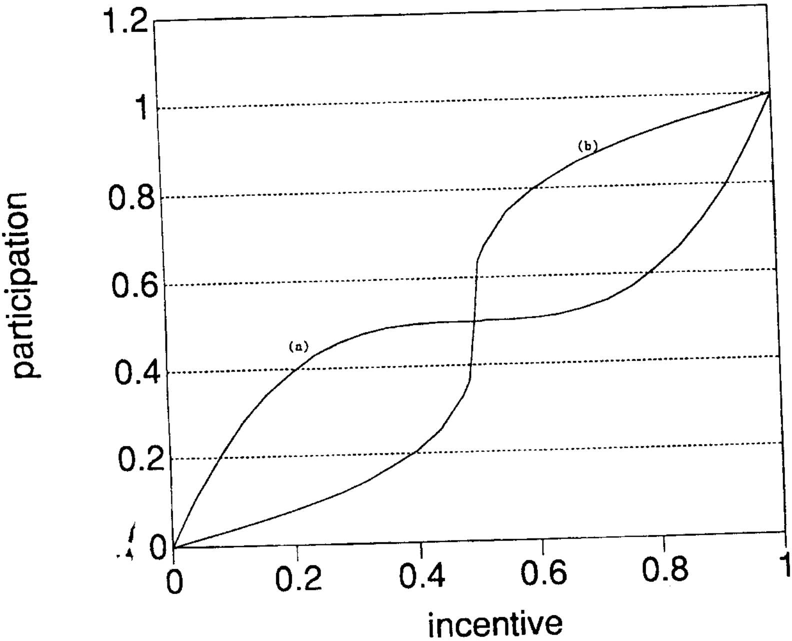

FIGURE 4. More complex participation functions. (a) Decreasing, then increasing returns: $F(x) = (1/2)((2x-1)^3 + 1)$. (b) Increasing, then decreasing returns: $F(x) = (1/2)((2x-1)^{1/3} + 1)$. See Figure 1 for the definition of terms.

4. BEHAVIOR OF THE MODEL WITH COMBINED DECREASING AND INCREASING RETURNS

A participation function could be convex in one interval of incentives and concave in another (or even more complex). For example, bandwagon effects might lead to a convex participation function for incentives below a certain threshold, while increasing resistance of wealthy couples might lead to a concave participation function for incentives above that threshold. Such a participation function would be S-shaped, like the logistic curve or the cumulative distribution function of the normal distribution. An S-shaped participation function could (but need not necessarily) have the intuitively appealing property of being differentiable at the boundaries of both low and high incentives. Alternatively, as incentives increased from low to high, decreasing returns might precede increasing returns. Some numerical calculations suggest that these possibilities yield few surprises in the dynamic behavior of program participation according to (2.4).

Figure 4 plots two forms of the participation function

$$F(x) = \tfrac{1}{2}((2x-1)^q + 1), \qquad 0 \le x \le 1, \quad 0 < q < \infty \tag{4.1}$$

for $q = 1/3$ (convex for $x < 1/2$, concave for $x > 1/2$) and $q = 3$ (concave for $x < 1/2$, convex for $x > 1/2$). The function (4.1) with $q = 1/3$ has derivative 1/3 at $x = 0$ and $x = 1$, so is S-shaped without being differentiable at the boundaries of x. It

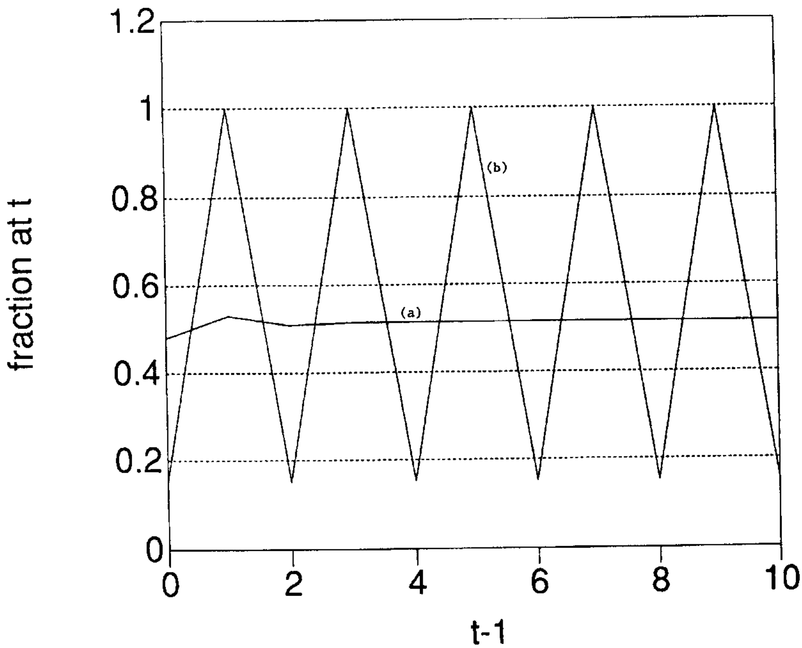

FIGURE 5. Trajectories in time, or forward orbits, of the program participation rate according to (2.4) and (4.1) with budget $c = 1/3$. (a) Decreasing, then increasing returns ($q = 3$). (b) Increasing, then decreasing returns ($q = 1/3$).

follows from the appendix that there is a unique fixed point of the iteration function G defined in (2.4).

Trajectories are computed by using the two forms of the participation function (4.1) with $q = 1/3$ and $q = 3$ in (2.4). When the dimensionless per capita budget c is less than 1/2, the participation rate oscillates between two values if $q = 1/3$, but rapidly converges to a fixed value if $q = 3$. These behaviors are illustrated in Figure 5 for $c = 1/3$. This behavior is just what would be expected from the results of Section 3 if the convex (respectively, concave) portion of the participation function for $x < 1/2$ were all that mattered. In reverse, when the dimensionless per capita budget c exceeds 1/2, the participation rate rapidly converges to a fixed value if $q = 1/3$, but oscillates between two values if $q = 3$. This behavior is just what would be expected from the results of Section 3 if the concave (respectively, convex) portion of the participation function for $x > 1/2$ were all that mattered. When $c = 1/2$, the trajectory for $q = 1/3$ is identical to that for $q = 3$; in both cases, the participation rate alternates between 1/2 and 1.

The chief difference between the trajectories determined by the participation function (4.1) and the trajectories determined by the participation function (2.5) seems to be that, under (4.1), the dimensionless per capita budget c affects qualitatively the form of the trajectories. A change in c can change two-point oscillations to convergence toward a fixed point, or vice versa. It is possible to interpret this finding in terms of the difference Bongaarts and Greenhalgh (1985) anticipate between the dynamics of participation in localities with low versus high per capita budgets. For the participation function with decreasing returns for low incentives and increasing returns for high incentives ($q = 3$ in (4.1)), a low dimensionless per capita budget ($c < 1/2$) produces convergence to a fixed point, while a high dimensionless per capita budget ($c > 1/2$) produces persistent oscillations.

5. BEHAVIOR OF THE MODEL WITH LEARNING FROM CUMULATIVE EXPERIENCE

To assume that the participation of prospective parents depends on the most recent incentive only, and not on any earlier incentives, is probably unrealistic. Prospective parents must estimate the costs and benefits of having, or not having, a child over many years. They may be less eager to participate in a one-child program after a high incentive is offered if they know that, in earlier years, anticipated benefits have sometimes not been realized.

We now model one way that parents might use more than one year's information about prior incentives to adjust their current level of participation. Suppose that parents respond to a weighted average of all previous incentives. Specifically, suppose there is a weighting coefficient w, $0 \leq w \leq 1$, such that last year's incentive x_{t-1} is weighted by w relative to this year's incentive x_t, and the incentive x_{t-2} of two years ago is weighted by w^2 relative to this year's incentive, and so on for earlier years. The weighted average incentive up to and including year t is defined by

$$y_0 = x_0 = c,$$
$$y_t = \frac{x_t + w x_{t-1} + w^2 x_{t-2} + \cdots + w^t x_0}{1 + w + w^2 + \cdots + w^t}, \qquad t \geq 1. \tag{5.1}$$

At one extreme, if $w = 0$, then $y_t = x_t$, i.e., the parents consider only the current year's incentive, as in previous sections. At the opposite extreme, if $w = 1$, then parents weigh the current and all prior incentives equally. (In principle, parents might weigh more remote incentives *more* than more recent incentives, perhaps because results in the early days of the program were impressed on their memories, but this possibility will receive no further attention here.) Now suppose that participation depends on the weighted average incentive according to

$$p_{t+1} = F(y_t), \tag{5.2}$$

instead of $p_{t+1} = F(x_t)$ as previously. We shall investigate the participation function $F(x) = x^q$ as in (2.5); obviously the alternative (4.1) could also be studied, but will not be here. Finally, suppose that the incentive depends on the participation according to

$$x_t = \min\left(1, \frac{c}{p_t}\right), \qquad x_0 = c, \qquad t = 1, 2, \ldots \tag{5.3}$$

which is a dimensionless form of (2.2). These assumptions, together with values for the three parameters c, q and w, completely specify the dynamics of participation and incentives.

Since the right side of (5.3) cannot decrease when c increases, the current incentive, and therefore the cumulative average incentive as well as the participation rate at every time, also cannot decrease when c increases. Thus localities with relatively higher per capita budgets c have higher participation rates, all else being equal.

As in Sections 2 and 3, three cases will be considered: decreasing returns ($q = 1/2$), constant returns ($q = 1$), and increasing returns ($q = 2$). All the computations reported here for a low budget ($c = 1/3$) have been carried out also for a high budget ($c = 2/3$) with qualitatively identical results, which are omitted for brevity.

The behavior of this model is unexpectedly rich. To appreciate this richness, the reader is invited to try to guess how the model behaves before reading the following description.

When the participation function has decreasing returns ($q = 1/2$), then regardless of the weighting coefficient w, the trajectory of participation shows rapidly damped oscillations and tends in the limit of large time towards the fixed point which is (in the absence of memory) the limiting value p^* given by (7.6). The numerical evidence in support of the mathematically unproved assertion that the limit of participation p_t is p^* is illustrated by the following example. When $w = 0.1$ and when $w = 0.5$, $p_{50} = 0.69336127435063$. According to (7.6), $p^* = (1/3)^{1/3} = 0.69336127435063$, which agrees to the number of places calculated.

When the participation function has constant returns ($q = 1$), then the trajectories show two qualitatively different modes of behavior. When $w = 0$, the participation follows a two-point cycle given by (7.5). When $w > 0$, the participation shows damped oscillations and tends in the limit of large time towards the fixed point p^* given by (7.2). Any amount of memory ($w > 0$) qualitatively changes the effect of constant returns from periodic cycling to asymptotically constant participation. The numerical evidence in support of the mathematically unproved assertion that, when $w > 0$, the limit of participation p_t is p^* is illustrated by the following example. When $w = 0.01$, the values of p_t for $t = 495, 496, \ldots, 500$ are 0.57733687057341, 0.57736340014135, 0.57733740115258, 0.57736288014987, 0.57733791072128, and 0.57736238075050. Even after 500 time steps, damped oscillations are visible in the fifth decimal place. According to (7.6), $p^* = (1/3)^{1/2} = 0.57735026918963$, which falls within the range of oscillation. When $w \geq 0.1$, p_t effectively converges to p^* by $t = 20$.

When the participation function has increasing returns ($q = 2$), then the trajectories show two qualitatively different modes of behavior, depending on w. There is a critical w^*, which appears to be near 1/3, such that if $w < w^*$, the participation converges to a two-point cycle. The amplitude of this cycle decreases as w increases from 0 to w^*. This behavior is illustrated by the trajectories shown in Figure 6(a, b) for $w = 0.1$ and $w = 0.3$, respectively. If $w > w^*$, the participation shows damped oscillations and tends in the limit of large time towards the fixed point p^* given by (7.2). This behavior is illustrated by the trajectory shown in Figure 6(c) for $w = 0.5$. The numerical evidence in support of the mathematically unproved assertion that, when $0 \leq w < w^*$, the participation converges to a two-point cycle (rather than merely being very slowly damped toward a unique limit) is illustrated by the following example. When $w = 0.01$, the values of p_t for $t = 495, 496, \ldots, 500$ are 0.11854590316418, 0.98692909273171, 0.11854590316418,

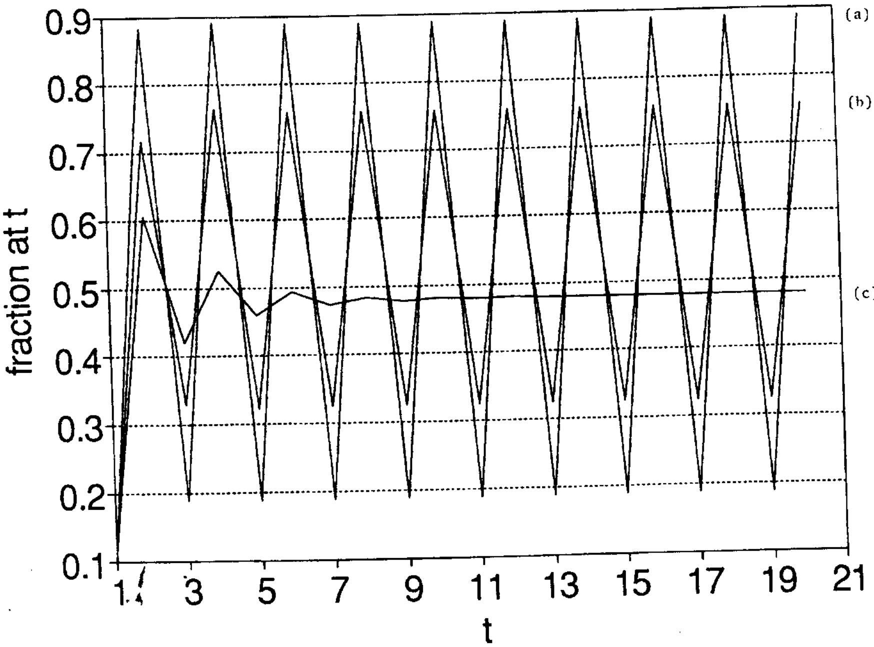

FIGURE 6. Trajectories in time, or forward orbits, of the program participation rate when prospective parents learn from cumulative experience ((5.1)–(5.3)) with budget $c = 1/3$ and increasing returns to incentives ($q = 2$ in (2.5)). (a) Weight $w = 0.1$: the participation settles into a two-point cycle. (b) Weight $w = 0.3$: the participation settles into a two-point cycle of smaller amplitude than that of the previous case. (c) Weight $w = 0.5$: following damped oscillations, the participation approaches a limit $p^* = (1/3)^{2/3}$ which is the fixed point of the participation function.

0.98692909273171, 0.11854590316418 and 0.98692909273171, which are exactly cyclic to the number of places computed. Similarly, when $w = 0.1$, the values of p_t for $t = 495, 496, \ldots, 500$ are 0.18624999668722, 0.88954454353336, 0.18624999668722, 0.88954454353336, 0.18624999668722 and 0.88954454353336, again exactly cyclic to the number of places computed. The numerical evidence in support of the mathematically unproved assertion that, when $w > w^*$, the limit of participation p_t is the fixed point p^* is illustrated by the following example. When $w = 0.5$, the values of p_t for $t = 90, 91, \ldots, 100$ are all equal to 0.48074985676914. According to (7.2), $p^* = (1/3)^{2/3}$, which is numerically identical to the number of places calculated. (For a model very similar to that in (5.1)–(5.3), Cohen and Newman, in preparation, have proved that the critical value of w^*, when $F(x) = x^q$, is $(q-1)/(q+1)$, which is 1/3 when $q = 2$.)

The participation rate at $t = 199$ and $t = 200$ according to the model with learning from cumulative experience is shown in Figure 7, for all values of the weight w. The participation shown in Figure 7 differs from what is believed to be the limiting behavior of participation only for constant returns and small positive w (Figure 7(b), extreme left), where the participation has not yet converged to the fixed point p^*.

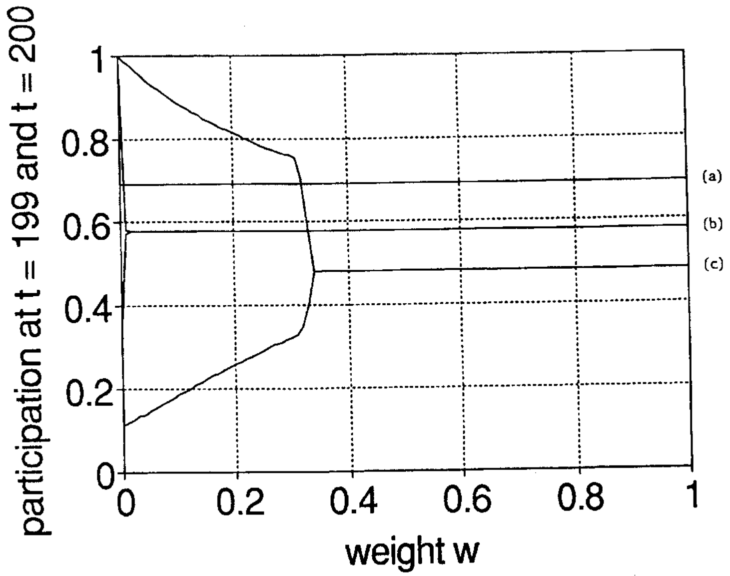

FIGURE 7. The program participation rates p_{199} and p_{200} at $t = 199$ and $t = 200$ according to the model of learning from cumulative experience ((5.1)–(5.3)) with budget $c = 1/3$. Values are computed for w from 0 to 1 by increments of 0.01. (a) Decreasing returns ($q = 1/2$ in (2.5)): p_{199} and p_{200} are indistinguishable from each other and from $p^* = 0.69336$. (b) Constant returns ($q = 1$ in (2.5)): for $w = 0$, p_{199} and p_{200} fall on a two-point limit cycle. For all $w > 0$, as $t \to \infty$, p_{2t-1} (the lower curve) and p_{2t} (the upper curve) approach a limit indistinguishable from each other and from $p^* = 0.57735$; but for small w, convergence to p^* is so slow that p_{199} and p_{200} differ visibly from p^*. (c) Increasing returns ($q = 2$ in (2.5)): for $w < w^* \approx 0.37$, p_{199} (the lower curve) and p_{200} (the upper curve) fall on a two-point limit cycle. Above w^*, p_{199} and p_{200} are indistinguishable from each other and from $p^* = 0.48075$.

To summarize, I compare how participation evolves when prospective parents learn from cumulative experience ($w > 0$) with how participation evolves when prospective parents do not ($w = 0$; Sections 2 and 3), according to the available numerical evidence. Under decreasing returns, the addition of learning from cumulative experience makes no difference to the long-run behavior of participation: the participation converges to the same unique fixed point as in the absence of learning. Under constant returns, the addition of learning from cumulative experience makes participation behave as if there were decreasing returns, i.e., it converges to the unique fixed point. Under increasing returns, there is a bifurcation at a critical value w^* of the weight attached to prior experience. For $0 \leq w < w^*$, participation settles into a two-point cycle. The larger w, the smaller the amplitude of this cycle. For $w > w^*$, participation converges to the fixed point p^*. In light of the calculations after (7.8), it appears that, for constant and increasing returns, when the participation converges to a single limiting value p^*, that limiting value is not in general equal to the long-term average participation $\overline{p}$ of the corresponding model without learning from cumulative experience. Thus, under constant or increasing returns, when participation approaches a unique limit, the participation is qualitatively (asymptotically constant instead of cycling) and quantitatively (p^* instead of $\overline{p}$) different from the participation in the absence of learning.

6. QUESTIONS ANSWERED AND RAISED BY THE MODEL

Bongaarts and Greenhalgh's (1985) account of a dynamic process in China's one-child program raises several questions which can be answered under the assumptions of the models just described.

Dynamics of Participation

How does the program participation rate differ between localities with different levels of economic incentives, all else being equal? The long-run average participation will be higher, the higher the per capita budget. If the participation function is inflected, as in Section 4, the program participation rate of the locality with the higher incentive may oscillate between two values while that of the locality with the relatively lower incentive may converge to a fixed value, or the reverse.

If many couples join a program, and the benefits per capita are diluted, what will happen next? As Bongaarts and Greenhalgh (1985) stated, the participation rate will fall in the next time period. The models provide additional details. If there are decreasing returns in participation, the subsequent fall will be less than the preceding rise, and the alternating rises and falls will gradually damp out as the participation rate approaches a steady limit. If there are constant returns in participation, the subsequent fall will just equal the preceding rise, and an undamped alternation between high and low values will ensue. If there are increasing returns in participation, alternating rises and falls in participation will adjust the participation to approach a single pair of alternating high and low values. If the participation function is inflected, as in Section 4, the mode of behavior depends on the per capita budget. If prospective parents average all prior incentives, as in Section 5, participation will converge to a steady limit under decreasing and constant returns in participation, as well as under increasing returns if prospective parents attach sufficient weight to earlier incentives. If there are increasing returns in participation but prospective parents attach less than a certain critical weight to earlier incentives, the participation will approach a pair of alternating high and low values. The difference between the limiting high and low values will be smaller, the higher the weight prospective parents attach to earlier incentives.

If couples drop out of a program because the expected benefits fail to materialize or are diluted, will the resources made available by the departure of those couples cause a subsequent increase in participation? Yes, as just described, and the fluctuations in the participation rate will persist or disappear depending on whether there are, on the one hand, constant or increasing returns or, on the other, decreasing returns in the participation function. The difference between persistent cycles of boom and bust versus an ultimately steady plateau in participation is determined, according to these models, by the concavity or convexity of the participation function in the region of the participation rate determined by the available per capita budget. Prospective parents' memory of earlier incentives may eliminate cycles in participation that would persist in the absence of learning from experience.

A referee remarked: "With respect to the one-child family program, however, it ought to be observed that local and higher level authorities, having observed such oscillations, would surely modify the program so as to attenuate them." That might not necessarily be the best thing to do, I say. While the models considered here are admittedly overly simple cartoons of China's one-child program, under some conditions the cycles they predict are probably socially superior to steady levels of participation and incentives. This tentative claim is based on analogy to Chu's (1990) game-theoretic model of individual decisions to obey or violate laws in traffic encounters. Without repeating his detailed argument, I will spell out the analogy between a program of fertility incentives and a program of traffic law enforcement, and then describe Chu's conclusions.

In a fertility incentive program and a traffic enforcement campaign, each individual (couple or solo) must trade off a preferred behavior (having another child, not waiting at a red light) against a socially provided incentive against that behavior (payments or fines). The expected incentive depends on the fraction of people who choose the preferred behavior (as more people reject the one-child incentives, those incentives rise for the acceptors; as more people run red lights, the likelihood of a traffic crackdown or "sweeping traffic rectification" increases). The function that relates participation at one time period to participation at another may be linear, convex, concave, or mixed; though independently derived, Chu's Figure 2 is identical in form to our Figure 1 plus Figure 4(b). Chu assumes that the fines collected by the government from traffic tickets are paid back to the public, so that they represent a private but not a collective cost; changing the sign of the fines gives incentives paid by a locality from general tax revenues. Chu gives a numerical example in which the total discounted social benefit under any steady enforcement policy is always less than the total discounted social benefit under a two-point cyclical policy. Though an analogous example for fertility incentives remains to be worked out, it seems very likely that, at least under some circumstances, a cyclical policy could be socially preferable to a steady one.

Empirical Questions

The models raise numerous empirical questions. Does the participation function of a real locality display decreasing, constant or increasing returns? Or does the participation function of a single locality display some combination of decreasing, constant and increasing returns over different ranges of incentive? Are the participation functions of different localities the same? Are the participation functions of different demographic, social or economic subgroups within a locality the same? (A priori, one might expect a nulliparous 20-year-old wife to be more easily persuaded not to have a child during the coming year than a nulliparous 37-year-old wife. Similarly, one might expect a woman with wealthy relatives to be less attracted by a given cash incentive than another woman of the same age with poor relatives.) Does the participation function of a given locality remain constant in time? All of these questions presuppose an answer to a methodological question: How can a participation function be estimated from experimental or observational data?

Do localities really spend all their available budget on the participating couples, subject to an upper limit on the per capita incentive, as assumed in (2.3) and (2.4)? If so, is the upper limit (b in (2.3)) the same from one locality to another? Otherwise, what is the connection between the participation rate in one period and the incentive announced or perceived at the beginning of the next (i.e., what is the replacement for (2.2))? If a locality does not expend all its budget in one time period, does it lose the surplus (as the model assumes) or is it able to cumulate the surplus against future need? How much attention do prospective parents pay to earlier incentives?

Alternative Models

The answers to each empirical question may suggest other models with different assumptions from those made here. Some possible alternatives are listed in the remainder of this section. I do not venture to guess the behavior of these alternatives. Each one requires its own analysis or computation.

A multi-locality model might consider the aggregate participation rate from a few or many localities that have different participation functions, different budgets, or different ceilings on the per capita incentive. A model with local heterogeneity might consider the impact of having different subgroups that respond differently to the same incentive. The assumption that the available budget is uniformly distributed among all participants might be replaced by an assumption that couples who sign up for the program receive a prescribed stream of benefits as long as they remain in the program (a suggestion of the referee). Alternatively, the incentive offered could be directly or inversely proportional to a couple's other income.

A time-inhomogeneous model might allow for secular trends or stochastic fluctuations in the values of the iteration rule's parameters, such as the population size and the budget. Different assumptions about the transferability in time of any unused budget could be modeled by considering, e.g., growth from the investment of savings versus depreciation from the aging of housing or food stocks. The budget could be made to depend positively or negatively on the fraction of couples in the one-child program in prior periods, inducing long-run feedbacks in participation (Ronald D. Lee, personal communication, 13 January 1992). Instead of assuming that the fraction of participating couples is a continuous variable, the number of participating couples could be modeled as a discrete variable. Other models of parental memory for prior incentives could be considered, e.g., a finite memory span for earlier incentives. Models of wishful (or fearful) thinking might assign more (or less) weight to higher incentives than to lower incentives.

The model analyzed here assumes that a couple's decisions to enter and to leave the program are both reversible. A different class of models could assume that either or both of these decisions is or are irreversible. For example, in a voluntary sterilization program, an individual once sterilized cannot (easily) be unsterilized, so the decision to enter the program is irreversible. A couple that leaves the one-child program by bearing a second child leaves the program irreversibly. The model analyzed here could be interpreted as applying to the proportion of eligible couples in the locality, rather than to all the couples in the locality, in order to allow for such irreversible departures, but this reinterpretation makes the assumption of a time-invariant participation function less plausible; it would be better to model irreversible departures directly. Because there may be a delay between conception and the appearance of pregnancy, it might be of interest to model a latency between the actual withdrawal from a program and the recognition of withdrawal by the locality or by other couples.

CONCLUSION

Modeling the dynamics of participation in fertility incentive programs confirms some intuitive expectations (Bongaarts and Greenhalgh 1985). Modeling also generates a more detailed picture of the conditions under which these expectations are fulfilled and generates a rich set of further empirical and theoretical questions. The implications of the modeling for policy, and for the comparison of centralization versus local control in family planning programs, will be explored elsewhere (Cohen and Newman, in preparation).

APPENDIX: ANALYSIS OF THE MODEL WITH DECREASING, CONSTANT, OR INCREASING RETURNS

This appendix presents a partial analysis of the general model (2.4) and a detailed analysis of the specific model (2.6). Standard terms from the theory of dynamical systems defined in Devaney (1989) will not be defined here. The agenda is first to describe the fixed points, then the 2-point cycles, and finally the forward orbits of these models.

Fixed points. Assume the participation function F is a continuous, strictly increasing function from $[0,1]$ onto $[0,1]$ such that $F(0)=0$ and $F(1)=1$. Define $G(p) \equiv F(\min(1, c/p))$ as in (2.4). (Examples of $G(p)$ are graphed in Figure 2.) We now prove that $G(p)$ has exactly one fixed point $p^* = G(p^*)$ in $[0,1]$, and that fixed point p^* lies in $(c,1)$. Since $\min(1,c/p)$ is continuous and weakly decreasing in p, G is also continuous and weakly decreasing in p. Where $\min(1,c/p)$ is strictly decreasing in p, G is also strictly decreasing in p. For $p \leq c$, $G(p) = F(1) = 1$. For p in $(c,1)$, $\min(1,c/p)$ is strictly decreasing in p, and $G(1) = F(q) < 1$. Therefore $G(p) - p$ is positive for $p \leq c$ and is negative for $p = 1$ and is strictly decreasing on $(c,1)$. Hence for exactly one p in $(c,1)$, $G(p) - p = 0$. This proves the claim.

Now assume F is differentiable on $[0,1]$. Then for $p \neq c$,

$$G'(p) = F'\left(\min\left(1, \frac{c}{p}\right)\right) \frac{d}{dp} \min\left(1, \frac{c}{p}\right). \tag{7.1}$$

If $p < c$, then $(d\min(1,c/p))/dp = 0$ so $G'(p) = 0$. If $p > c$, then $(d\min(1,c/p))/dp = d(c/p)/dp = -cp^{-2}$ so $G'(p) = -F'(c/p)cp^{-2}$.

Now assume (2.5). On the interval $(c,1)$ where the fixed point p^* lies, $G(p) = (c/p)^q$. To find p^*, we set $G(p) = p$ and obtain

$$p^* = c^{q/(1+q)}, \qquad G'(p^*) = -q. \tag{7.2}$$

Thus p^* is an increasing function of c and a decreasing function of q. Moreover, p^* is an attracting fixed point if and only if $q < 1$, and p^* is a repelling fixed point if and only if $q > 1$. If $q = 1$, p^* is neutrally stable.

Two-point cycles. For the analysis of 2-point cycles, we again begin with the general model (2.4). We prove that, in $[0,c]$, there exists at most one participation

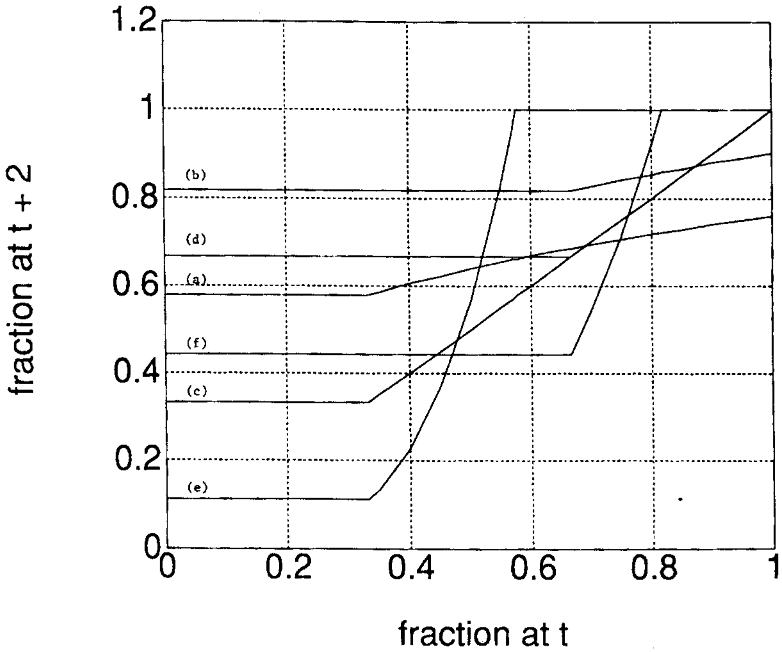

FIGURE 8. The second iterate of each iteration rule (2.6) graphed in Figure 2 predicts the participation rate two years hence as a function of the participation rate in the present year, according to $p_{t+2} = G(G(p_t))$ with (a) decreasing returns ($q = 1/2$) and low budget ($c = 1/3$); (b) decreasing returns ($q = 1/2$) and high budget ($c = 2/3$); (c) constant returns ($q = 1$) and low budget ($c = 1/3$); (d) constant returns ($q = 1$) and high budget ($c = 2/3$); (e) increasing returns ($q = 2$) and low budget ($c = 1/3$); and (f) increasing returns ($q = 2$) and high budget ($c = 2/3$).

rate p, say p_2, with period 2. For all $p \le c$, we have $1 \le c/p$ so $G(p) = F(1) = 1$, hence $G(G(p)) = G(1) = F(\min(1,c)) = F(c)$. So any point $p \le c$ of period 2 satisfies $p = G(G(p)) = F(c) \le c$. So a participation rate $p_2 \le c$ with period 2 exists if and only if $F(c) \le c$, and in that case $p_2 = F(c)$. This proves the claim.

The argument just given also establishes that if a participation rate $p_2 \le c$ with period 2 exists, then $G(p_2) = 1$, i.e., the other point on the 2-cycle that contains p_2 is the participation rate 1.

Now assume (2.5). Then it is easy to check that

$$G(G(p)) = \begin{cases} c^q & p \in [0,c], \\ \dfrac{p^{q^2}}{c^{q^2-q}} & p \in [c, \min(1, c^{1-1/q})], \\ 1 & p \in [\min(1,c^{1-1/q}), 1]. \end{cases} \tag{7.3}$$

The interval $[\min(1,c^{1-1/q}),1]$ is empty if $q < 1$, contains just the point 1 if $q = 1$, and has positive length if $q > 1$. $G(G(p))$ is graphed in Figure 8 for the same parameter values as in Figure 2.

Any point of intersection of the line of slope 1 through the origin with the graph of $G(G(p))$ satisfies $p = G(G(p))$ and hence is a periodic point of G with period

2. On the interval $[0,c]$, there are no points of period 2 if $0 < q < 1$ since then $F(c) > c$, while if $q \geq 1$, the unique point of period 2 in $[0,c]$ is $p_2 = F(c) = c^q$. The other point on the 2-cycle that contains p_2 is 1.

On the interval $(c,1]$, if $q = 1$, every point in $[c,1]$ has period 2, and every point in $[c,1]$ except p^* (which has prime period 1) has prime period 2. To prove this, let $q = 1$, $p \neq p^*$, and $c \leq p \leq 1$. Then $G(p) = c/p \leq 1$ and $G(G(p)) = \min(1, c/(c/p)) = p$. Thus p has period 2, and p has prime period 2 because only p^* has period 1.

The point $p = 1$ has period 2 if and only if $q \geq 1$, as is clear from (7.3). On $(c,1)$, when $q \neq 1$, $G(G(p)) = p < 1$ if and only if $[c/(c/p)^q]^q = p$, i.e., if and only if $p = p^*$. So there are no points of prime period 2 on $(c,1]$ other than $p = 1$, and $p = 1$ has prime period 2 if and only if $q \geq 1$.

In short, if $q < 1$, G has no points of prime period 2. If $q = 1$, then every p in $[c,1]$ has prime period 2 except p^*, which has prime period one. If $q > 1$, there is a unique 2-cycle (aside from the trivial one on p^*), namely, the 2-cycle through the points p_2 and 1.

If $q = 1$, the points in $[c,1]$ are neutrally stable, because then $G(G(p)) = p$ so $dG(G(p))/dp = 1$. If $q > 1$, the only point of prime period 2 in $[c,1]$, namely, $p = 1$, is an attracting periodic point, because $dG(G(p))/dp = 0$.

Forward orbits. Finally, we describe analytically the forward orbits of (2.6), which are plotted numerically in Figure 3. For the special initial condition given in (2.6), when $q < 1$, direct computation shows that

$$p_1 = c^q, p_2 = c^{q(1-q)}, p_3 = c^{q(1-q+q^2)}, p_4 = c^{q(1-q+q^2-q^3)}, \ldots$$
$$p_n = c^{q\sum_{i=0}^{n-1}(-q)^i}, \tag{7.4}$$
$$\lim_{n\to\infty} p_n = c^{q/(1+q)}.$$

The limit of p_n is just the attracting fixed point p^* given by (7.2).

When $q \geq 1$,

$$p_{2n+1} = c^q, \qquad p_{2n+2} = 1, \qquad n = 0,1,2,\ldots. \tag{7.5}$$

For arbitrary initial conditions x_0 in $[0,1]$, the forward orbits of (2.6) may be described by using graphical analysis to obtain phase portraits (Devaney 1989, pp. 20–21). When $q < 1$, regardless of x_0,

$$\lim_{n\to\infty} p_n = c^{q/(1+q)} = p^*. \tag{7.6}$$

When $q = 1$ and $x_0 \leq c$, then $p_1 = x_0$, and for $n > 1$, p_n is identical to that in (7.5) (taking $q = 1$ there); when $c < x_0 \leq 1$,

$$p_{2n+1} = x_0, \qquad p_{2n+2} = \frac{c}{x_0}, \qquad n = 0,1,2,\ldots. \tag{7.7}$$

It follows, as has previously been noted, that when $q = 1$, every point in $[c,1]$ lies on a periodic orbit of period 2; except for the orbit that is constant at p^*, these period-2 orbits have prime period 2.

When $q > 1$, if $p_1 = (x_0)^q$ falls in $[0, c]$, then for $n \geq 1$, p_n is identical to that in (7.5) (taking $q > 1$ there); if $p_1 = (x_0)^q$ falls in $[c^{1-1/q}, 1]$, then the orbit is the 2-cycle given by (7.5) (taking $q > 1$ there), though with the opposite phase (i.e., n is shifted by one). When $q > 1$ and $p_1 = (x_0)^q$ falls in $[c, c^{1-1/q}]$, then, according to the available numerical results, the forward orbit of p_n oscillates more and more widely, rapidly approaching the 2-cycle on the values c^q and 1, where it remains. It can be proved with little difficulty that $dG(G(p))/dp > 1$ throughout $[c, c^{1-1/q}]$ if and only if $dG(G(p))/dp\,|_{p=c} > 1$, and since $dG(G(p))/dp\,|_{p=c} = q^2 c^{q-1}$, this is true if and only if $c > q^{-2/(q-1)}$. The last inequality is, of course, satisfied by $q = 2$, $c = 1/3$ or $c = 2/3$, as in the numerical illustration. When $dG(G(p))/dp > 1$ throughout $[c, c^{1-1/q}]$, the orbit of p must leave the interval $[c, c^{1-1/q}]$ after some finite number of iterations of G. What remains unproved at present, but appears to be true from numerical calculations not presented here, is that the orbit of p leaves $[c, c^{1-1/q}]$ after a finite number of iterations of G, even when $c \leq q^{-2/(q-1)}$.

The long-run average participation rate of a forward orbit, assuming the following limit exists, is

$$\overline{p} = \lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} p_n. \tag{7.8}$$

Assuming (2.5), the limit always exists and

$$\overline{p} = \begin{cases} p^*, & q < 1, \\ \frac{1}{2}\left(p + \frac{c}{p}\right) \quad \text{for} \quad 0 \leq p \leq c, & q = 1, \\ \frac{1}{2}(1 + c^q), & q > 1. \end{cases} \tag{7.9}$$

It is easy to prove that $\overline{p} \geq p^*$ if $q = 1$ but that $\overline{p}$ may be greater or less than p^* if $q > 1$. For example, in Figure 3, in orbit (e), $(1 + (1/3)^2)/2 = (1 + 0.11)/2 = 0.56 > 0.48 = (1/3)^{2/3}$ but, in orbit (f), $(1 + (2/3)^2)/2 = (1 + 0.44)/2 = 0.72 < 0.76 = (2/3)^{2/3}$.

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