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GRAPH THEORETIC MODELS OF FOOD WEBS

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ABSTRACT. A food web is a directed graph that tells which kinds of organisms nourish which other kinds of organisms in a community of species. Each vertex in such a graph corresponds to a kind of organism. Each arrow or directed edge between vertices corresponds to a flow of energy or biomass from one kind of organism to another.

Another description of communities of species represents each kind of organism by a multidimensional hypervolume in a hypothetical ecological niche space. In niche space, each dimension corresponds to some environmental variable or some variable characterizing the food consumed by the organisms. The multidimensional hypervolume associated with each kind of organism is called its niche. The projection of ecological niche space onto the dimensions characterizing the food consumed is called trophic niche space, and the same projection of a niche is called the trophic niche.

An elementary question about niche space is: what is the minimum dimensionality of a niche space necessary to represent, or to describe completely, the overlaps among observed niches? This question remains unanswered for niche space in general. We have developed a technique for answering it for trophic niche space.

If the trophic niche of a kind of organism is a connected region in trophic niche space, then it is possible for trophic niche overlaps to be described in a one-dimensional space if and only if the trophic niche overlap graph is an interval graph. The trophic niche overlap graph is an undirected graph in which the vertices are the predators in a food web and two predators are joined by an undirected edge when there is some kind of prey that both predators eat.

An analysis of 30 food webs using the combinatorial theory of interval graphs suggests that a niche space of dimension one suffices, with unexpectedly high frequency and perhaps always, to describe the trophic niche overlaps implied by real food webs in single habitats. In order to give a precise quantitative meaning to the phrase, "unexpectedly high frequency," the proportion of interval graphs in each of seven model universes was estimated by a combination of analysis and Monte Carlo simulations separately for each food web. It appears that real food webs fall in a small subset of the set of mathematically possible food webs. That real food webs are compatible with one-dimensional

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niche spaces, more often than can be explained by chance alone, has not been noticed previously.

Other findings about the structure of food webs emerged from the analysis of the machine-readable collection of food webs. In food webs which describe a community, the ratio of the number of kinds of prey to the number of kinds of predators is close to a constant 3/4, over all food webs. The mean and variance of the number of overlaps among the predators' diets are reasonably described by assuming that every kind of predator has a constant and independent probability of preying on every kind of prey, with a probability which is characteristic of the food web.

References

1. J. E. Cohen, *Food webs and the dimensionality of trophic niche space*, Proceedings of the National Academy of Sciences 74 (1977), 4533–4536.

2. ———, *Food Webs and Niche Space*, Princeton University Press, Princeton, 1978.

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