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JOEL E. COHEN, Kings College Research Centre, Cambridge

This paper (Cohen (1975)) establishes a new class of ergodicity theorems for the age structure of populations, and proposes applications.

Let $S = \{A_1, A_2, \cdots\}$ be a denumerable set of Leslie matrices $A_i$ satisfying the assumptions of the Coale-Lopez theorem of weak ergodicity for populations, and let $m(0)$, $n(0)$ be any two initial population age structures (column vectors) satisfying the assumptions of the Coale-Lopez theorem. Suppose $m(t)$ is the population age structure obtained at time $t$ by premultiplying $m(0)$ sequentially by $t$ elements of $S$, $A_1, \cdots$, where the sequence of matrices following $A_1$ is determined by a homogeneous irreducible aperiodic positive recurrent geometrically convergent Markov chain on the state space $S$; and suppose that $n(t)$ is independently determined by the same Markov chain in the same way, starting from $A_2$. Then (strong stochastic ergodicity) all moments of the two random variables $m_i(t)/m_i(t)$ and $n_i(t)$ and $n_i(t)/n_i(t)$ converge and they converge in distribution as $t \to \infty$. The same conclusion holds if (weak stochastic ergodicity) the Markov chain is finite and weakly ergodic (in the sense of Hajnal), but not necessarily homogeneous.