HOW SHOULD AN HONORARY learned society such as the American Philosophical Society (APS) regulate its size? This question, of obvious practical importance to the APS, is a special case of a broader scientific question: how is the size of any population, human or not human, regulated? In this broader perspective, the APS provides a valuable case study, a microcosm, of the self-regulation of a human population.

This article answers these questions: What has been the trend in the average age of a newly elected resident member? What has been the trend in the average age at death of a resident member? What has been the trend in the average duration of membership? These questions are steps en route to answering this question: How will future numbers of resident members to be elected annually affect the steady-state size of the APS resident membership? If the APS elects 35 resident members a year for the indefinite future (as it had in the 17 years preceding 2006), will there be an eventual steady-state size of the resident membership, and if so, how big will it be? What if APS elects 25 or 45 resident members a year for the indefinite future?

A reader who is interested only in the answers to these questions, and not in the data or analyses that led to these answers, is invited to skip to the section called “Conclusions.” The data, analysis, and conclusions are restricted to resident members. Henceforth, “members” will be taken to refer strictly to resident members.

DATA

At the end of 2005, the APS had 753 living resident members. In the 17 years from 1989 to 2005, the number of living resident members of the APS increased by more than 200.
On 10 March 2006, Nora Monroe of the Membership Office emailed me a list of all 1,776 people who were elected to membership in the APS from 1930 through 2005. The dates of birth and ages at election of some individuals were missing. By consultation with Wikipedia http://www.wikipedia.org/ and other online information sources, my assistant, Priscilla Rogerson, and I found the year of birth of all but four individuals. One other person resigned. This study was based on the remaining $1,771 = 1,776 - 4 - 1$ people. For each person in this analysis, the available data were the year of birth, the year of election, the age at election (given exactly by the individual or computed as the year of election minus the year of birth), and the year of death (taken as 0 if the individual was still alive as of 10 March 2006).

The number of living members on 10 March 2006, computed from this information, was 750. This information is exactly consistent with independent information furnished by Nora Monroe, which showed 753 living members at the end of 2005, because the data sent to me as of 10 March 2006 included three deaths during 2006: $753 - 3 = 750$. Hence $1,021 = 1,771 - 750$ members elected from 1930 through 2005 had died by 10 March 2006.

In the 17-year interval 1989–2005 inclusive, the APS elected 589 individuals to resident membership (an average of 34.65 members per year) and 381 of its resident members died (an average of 22.41 members per year). The excess of the number of elections over the number of deaths resulted in an increase of 208 resident members (an average increase per year of 12.24 members).

### Cohort Duration of Membership among Members Who Died

For members who had died by 10 March 2006, the “duration of membership” was defined as the year of death minus the year of election. For members still alive by 10 March 2006, no “duration of membership” was defined.

Figure 1 shows the duration of membership of all 1,021 deceased members by year of election. The downward sloping ramp of the largest duration of membership in each year of election from, approximately, 1944 to 2005 is a consequence of the fact that, for example, an individual elected in 1995 who is dead by 2006 cannot have a duration of membership longer than 11 years. Individuals elected from 1930 to about 1944 had durations of membership that ranged from 0 to 58 years.

Figure 2 shows the duration of membership among deceased members, by year of death from 1930 to 2006. The upward sloping ramp in the longest duration of membership for each year of death from 1930
to approximately 1989 is a consequence of the fact that, for example, an individual elected in 1930 who was dead by 1940 could not have been a member longer than 10 years.

The durations of membership of the 384 members who died from 1989 to 2006 (including three who died in 2006) were examined in greater detail (fig. 3). There was no evidence of an increasing or decreasing trend over this interval in the duration of membership among members who died 1989–2006. (Technically, a linear regression model was fitted in which the independent variable was the year of death and the dependent variable was the duration of membership. The squared correlation coefficient was 0.0008, not different from 0 statistically or substantively.) The average duration of membership of these members
was 26.02 years (from the year of election to the year of death). The minimum duration of membership of these members was 0 years (for members who died in the year of their election), the maximum was 58 years, the interquartile range was 20 years, and the standard deviation was 13.65 years.

Among the 381 members who died 1989–2005, the average age at death was 84.94 years and the average age at election was 58.95 years, giving an average duration of membership of 25.99 years. Whether or not the three members who died in 2006 were included in the calculation, the average duration of membership was very close to 26 years.

The average age at death among the 381 members who died 1989–2005 increased by 0.2600 year of age per calendar year over the interval 1989–2005. (Technically, a linear regression model was fitted in which the independent variable was the year of death and the dependent variable was the age at death. The 95% confidence interval of the estimated slope of 0.2600 ranged from 0.0954 to 0.4246.)

**Figure 3. Duration of membership among deceased members by year of death from 1989 to 2006**

**Period Duration of Membership in 1989–2005**

The average “period duration of membership” for 1989–2005 is defined as the average age at death of all individuals who died in 1989–2005 minus the average age at election of all individuals who were elected in 1989–2005. (The year 2006 is excluded from this analysis because the data were incomplete for 2006.) The difference between this so-called “period” analysis and the previous so-called “cohort” analysis is that in this “period” analysis, many of the 589 individuals who were elected 1989–2005 remained alive by the end of 2005 and
many of the 381 individuals who died 1989–2005 were elected prior to 1989.

Figure 4 shows individuals’ age at death (symbol −) as a function of year of death and age at election (symbol ▲) by year of election. The mean age at election of the 589 individuals elected in 1989–2005 was 64.05 years. The average age at election increased over this interval but, because of the variability in the age at election during these years, the evidence for an increase was not statistically significant. (Technically, a linear regression model was fitted in which the independent variable was the year of election and the dependent variable was the age at election. The 95% confidence interval of the estimated slope of 0.1596 ranged from −0.0014 to 0.3206, which did not exclude 0.)

From 1989 to 2005, the average age at death was 84.94 years, the average age at election was 64.05 years (substantially more than the average age at election of 58.95 years among the individuals who died 1989–2005), and the average period duration of membership was $20.89 = 84.94 - 64.05$ years (substantially shorter than the average cohort duration of membership of 25.99 years).

To display more clearly the main trends in the scattered data of figure 4, figure 5 shows the annual average age at death (symbol −), the annual average age at election (symbol ▲), and the annual period duration of membership (symbol ■) (defined as the difference between the annual average age at death and the annual average age at election), for each year from 1989 to 2005. No increasing or decreasing trend in the period duration of membership is evident. (Technically, a linear regression model was fitted in which the independent variable was the year from 1989 to 2005 and the dependent variable was the annual average...
period duration of membership. The squared correlation coefficient was 0.036, not different from 0 statistically or substantively.)

**Age at Election, Age at Death**

The results thus far pose a puzzle. The annual period duration of membership (symbol ■ in fig. 5) gives no statistically significant evidence of an increasing or decreasing trend in 1989–2005. Yet analyses above showed that, in 1989–2005, the average age at death among members who died in 1989–2005 increased statistically significantly while the average age at election increased in 1989–2005, but not at a rate that was statistically significantly different from 0. As the average period duration of membership is the average age at death minus the average age at election, it seems paradoxical to find that the average period duration of membership did not increase in 1989–2005, while the average age at death did increase and the average age at election did not increase.

To resolve this apparent discrepancy, note the substantial overlap in the 95% confidence intervals of the estimated annual rate of increase in the age at death and in the age at election. The annual increase in the age at death (technically, regression slope as a function of calendar year) had a 95% confidence interval from 0.0954 to 0.4246, while the annual increase in the age at election had a 95% confidence interval from -0.0014 to 0.3206. Both the average age at death and the average age at election could have increased annually, on the average, by any amount in the interval from 0.0954 to 0.3206. The evidence did not preclude

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**Figure 5.** For each year from 1989 to 2005, the annual average age at death (symbol ■) of individuals who died that year, the annual average age at election (symbol ▲) of individuals who were elected that year, and the annual average period duration of membership (symbol ■) (defined as the difference between the average age at death and the average age at election) in that year.
the possibility that both rates of increase were equal and the annual average period duration of membership was constant in 1989–2005.

A deeper understanding of this puzzle results from analyzing all the available data on age at election and age at death from 1930 to 2005. Figure 6 shows the age at election by year of election of the 1,771 members with complete information. Deceased and living members are included. The average age at election in a given year increased on average by 0.1697 year of age per calendar year from 1930 to 2005. (Technically, a linear regression model was fitted in which the independent variable was an individual’s year of election and the dependent variable was the individual’s age at election. The 95% confidence interval of the estimated slope of 0.1697 ranged from 0.1519 to 0.1875. This 95% confidence interval falls entirely within the overlap of confidence intervals from 0.0954 to 0.3206, which was pointed out above.) Over this 75-year interval, the average age at election in a given year increased by 12.7 years, according to this trend line. In round numbers, every six years, the average age of members elected increased on the average by one year. This increase in the average age of members at election slowed the increase in the number of members that would otherwise have resulted from the increasing longevity and greater numbers of members being elected.

Figure 7 shows the age at death by year of death of 1,021 members elected 1930–2005 who had died by the end of 2005. The average age at death in a given year increased by 0.2634 year of age per calendar year from 1930 to 2005. The average age at death in a given year increased
19.75 years over this 75-year interval, according to this trend line. In round numbers, every 3.8 years, the average age at death of members increased on the average by one year. This increase in the average age at death exceeded the annual increase in the average age at election by a statistically significant amount and added to the increase in the number of members that would otherwise have resulted from the greater numbers of members being elected. While the data (above) did not significantly reject the possibility that the period duration of membership was constant in 1989–2005, the data do significantly reject the possibility that the period duration of membership was constant from 1930 to 2005. In fact, the period duration of membership increased by $0.0937 = 0.2634 - 0.1697$ year per calendar year from 1930 to 2005. This rate of increase is equivalent to an increase in the period duration of membership of one year in just under 11 calendar years.

(Technically, a linear regression model was fitted in which the independent variable was an individual’s year of death and the dependent variable was the individual’s age at death. The 95% confidence interval of the estimated slope of 0.2634 year of age per calendar year ranged from 0.2341 to 0.2926. The estimate 0.2634 for 1930–2005 was very close to the estimate 0.2600 for 1989–2005. The confidence interval from 0.2341 to 0.2926 for 1930–2005 fell entirely within the confidence interval for 1989–2005. The lower end of the confidence interval, 0.2341, exceeded the upper end, 0.1875, of the confidence interval for the annual increase in the age at election, so age at death increased statistically significantly faster than the age at election from 1930 to 2005. A quadratic regression model was also fitted in which the independent
variables were an individual’s year of death and the square of the year of death [both centered to eliminate ill-conditioning], while the dependent variable was again the individual’s age at death. The coefficient of the square of the year of death, while slightly negative, did not differ significantly from 0, so the data gave no significant evidence that the rate of increase of the average age at death was slowing or accelerating from 1930 to 2005.)

In 1989–2005, the difference between the average cohort duration of membership, 25.99 years, and the average period duration of membership, 20.87 years, is 5.12 years. The rise in the age at election by year of election explains most of this difference. The members who died in 1989–2005 were elected about 26 years earlier on the average, and 26 years earlier the average age at election was lower by $26 \times 0.1697 = 4.4$ years.

Two projections of the average age at death of members elected in 1989–2005 are possible with the available data. If the long-term annual increase in the average age at death remains at 0.2634 year of age per calendar year, and if the average cohort duration of membership remains at 26 years, then the average age at death of members elected in 1989–2005 may be roughly 84.94 (the average age at death among members who died in 1989–2005) $+ 26 \times 0.2634 = 91.79$, or nearly 92 years. Alternatively, as the average age at election in 1989–2005 was 63.91 years, if the average cohort duration of membership remains at 26 years, then the average age at death of members elected in 1989–2005 may be roughly $63.91 + 26 = 89.91$ years, or nearly 90 years. Given the increasing trend of longevity, the higher estimate seems more plausible, but the difference between the projections is small.

Analysis of the Steady-State Population Size

Let us suppose that the APS establishes a fixed number of members who will be elected each year. This number includes those selected by the Class Committees, the temporary nominating groups, the Council, and any other, special procedure. Let us call this fixed number the “number elected.” If the number elected is held constant in future elections and if the average duration of membership remains constant in the future, then the number of members will approach a constant level determined by this simple equation:

$$\text{Number of members at steady state} = \text{average duration of membership} \times \text{number elected}.$$  

This equation applies to any population, not only APS members. For example, consider a committee in which the duration of appointment is
three years. Suppose two new members are elected each year. The above equation says that the steady-state population size of the committee is 3 (average duration of membership) $\times$ 2 (number elected per year) = 6. Why is this true? Suppose the committee starts with no members. In the first year, two members are appointed. In the second year, two additional members are appointed, bringing the committee membership to a total of four. In the third year, another two members are appointed, bringing the committee membership to a total of six. In the fourth year, the two original members drop off the committee and are replaced by another two members, maintaining the steady-state size of the committee at six. The same process of replacement continues in all future years. The committee will reach a steady-state size of six no matter what the initial number of members, because after at most three years all the initial members will leave the committee and the steady rotation of two members a year will take over.

This example shows that when a population is not at steady state, the average cohort duration of membership gives an accurate estimate of the steady-state population size, whereas the average period duration of membership gives a transient and incorrect estimate of the steady-state population size. In the example, in the first three years the period duration of membership is undefined because no one leaves the committee. The period duration of membership coincides with the cohort duration of membership (three years) only in the fourth year, when the committee size reaches steady state.

For the membership of the APS, which is far from steady state, the average cohort duration of membership was 25.99 years in 1989–2005 and 26.02 years in 1989–2006. In view of the increasing longevity of APS members, it is unlikely that the future mortality experience of the members elected 1989–2005 will coincide with the mortality experience of the members who died 1989–2005. Unless the average age at election rises as fast as the average age at death in the future, it is likely that the average cohort duration of membership based on 1989–2005 will underestimate the average cohort duration of membership in the future. Therefore, using the 1989–2005 cohort duration of membership to project a future steady state is likely to underestimate the future steady-state number of members of the APS.

The number elected is determined by the Council, the governing board of the APS. Figure 8 shows what the number of members at steady state would be if the cohort duration of membership remained constant at 26.02 years and if the Council fixed the number elected at any value ranging from 20 members per year to 60 members per year. If 29 members are elected each year, the membership will remain at 755 at steady state, again assuming no future change in the average
duration of membership. Each time the number elected increases by one per year for the indefinite future, the steady-state membership increases by about 26. For example, if the Council were to set the number elected at 33 members per year, the steady-state membership size would be 859 rather than 755.

The assumption of no future change in the average cohort duration of membership could fail if the average age at death of APS members continued to rise faster than the average age at election, or if (even in the absence of increased longevity) the average age at election were reduced as a matter of policy. If the average cohort duration of membership increases, then the APS Council and the membership will be forced to choose among several alternatives. The steady-state size of the membership could be allowed to increase while keeping constant the number elected annually, or the number elected could be decreased to keep the steady-state size of the membership constant, or some combination of adjustments in both the number elected and the steady-state size of the membership will be required, or some nominal age such as 85 or 90 could be adopted such that only the number of members at or below this threshold age is to be maintained below some desired membership size.

Conclusions

From 1989 through 10 March 2006, the average cohort duration of membership, defined as the number of years from the year of election to the year of death, of members of the APS was 26.02 years. If this dura-
tion of membership remains constant in the future, then fixing the number of members elected annually (by any procedure) at 29 will result at steady state in 755 members, which is approximately the 2006 size of the membership. Under the same assumption of constant duration of membership, electing 33 members each year will result in 859 members at steady state. Each additional member elected annually, year after year, increases the steady-state membership by just over 26 members. If APS elected 25 members each year, the steady-state membership would be 651; if 30 members, then 781; if 35 members, then 911; if 45 members, then 1,171.

From 1930 to 2005, the average age of members at their election increased by about 12.7 years, an increase of roughly 0.17 year of age per calendar year of time. The average age at death of members increased by about 19.8 years, an increase of roughly 0.26 year of life per calendar year of time.

If the average age at death of APS members continues to rise faster than the average age at election, or if the average age at election were reduced as a matter of policy, then the average duration of membership would increase. If the average duration of membership increases, then the above estimates of steady-state membership size will all be underestimates. In this case, the APS Council and the membership will be forced to permit the steady-state size of the membership to increase while keeping the annual number elected constant, or to decrease the annual number elected to keep the steady-state size of the membership constant, or to combine adjustments in both the annual number elected and the steady-state size of the membership, or to focus attention only on the number of members at or below some chosen age threshold.

Related Prior Studies of the APS and Other Learned Societies

Field (1897) analyzed the 1,118 deaths of (resident and foreign) members of the APS from 1743 to 1894, inclusive, where the ages at death were recorded. The average age at election was 47.49 years. The average age at death was 70.13 years, with a range from 25 to 101 years. The cohort average duration of membership was 22.64 years, with a range from six days to 70 years. Field also reported on the 314 deaths of resident (“American”) members who died during the 25 years ending 31 December 1894. The 293 members for whom information was available were elected at an average age of 48.57 years, lived an average of 21.34 years after election, and died at an average age of 69.91 years.

Wilks (1951) was asked by the Council of the American Philosophical Society “to estimate, under certain conditions, the average number
of new [resident] members to be elected per year during the next fifty years in order to maintain a [resident] membership of 500.” His information was limited to the number of members, the ages of the newly elected members, and the ages of the deceased members during the interval 1937–49. The year-end resident membership increased from 399 in 1937 to 494 in 1949. The average age at election was 54.5 years and the average age at death was 75.7 years, giving an average period duration of membership of 21.2 years. Wilks did not estimate cohort duration of membership. Using an ingenious mechanical Monte Carlo technique, Wilks projected “that a steady election rate of 24 new members a year (the average number of new members for the period 1937–1949) would produce a total membership which would rise to 560–570 by 1975 and then tend to stabilize by 1985 at 550–560.” (For comparison, we project that a steady election rate of 24 new members a year would produce a total membership that would stabilize at 624 or 625 members. The increased steady-state size compared with 550–560 projected by Wilks is due to the increase in longevity of APS members.)

Leridon (2004) investigated the demography of the French Académie des Sciences in great detail using the same basic equation as the present study. He noted (p. 99) that “the difference between the mean age of year \( n \) leavers and that of members elected in the same year [which is our average period duration of membership] . . . does not necessarily correspond to the mean length of stay of the leavers or to the future mean length of stay of the newly elected members, as the conditions of stationarity are not always met.” He reported (p. 97) that the cohort average duration of membership for the Académie Française fell from 22.7 years for members who died during 1700–99 to 16.7 years for members who died 1900–99, while the cohort average duration of membership for the Académie des Sciences fell from 21.0 years to 19.2 years for the same intervals.

A tension between a desire to keep a relatively young age structure in learned societies and a desire to limit the size of membership is widely perceived and is the subject of active research in several countries. On 30 November 2005, the Vienna Institute of Demography conducted a workshop on the demography of learned societies, which included presentations on the Académie des Sciences (Institut de France), 1666–2030, the Koninklijke Nederlands Akademie van Wetenschappen, 1802–2000, and the Austrian Academy of Sciences (Dawid et al. 2009; Feichtinger et al. 2007). The United States National Academy of Sciences has also conducted unpublished studies of its age structure and prospective size.

The demography of learned societies illustrates in microcosm the tradeoffs faced by larger societies. Steady-state population size, longevity, and the annual number of new members (usually called the number
of births per year in the broader context) are not independent, but are constrained at steady state, in the absence of migration, by the same equation given above.

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**References**


