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Interspecific competition affects temperature stability in Daisyworld

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ABSTRACT

The model of Daisyworld showed that nonteleological mechanistic responses of life to the physical environment can stabilize an exogenously perturbed environment. In the model, 2 species of daisies, black and white, stabilize the global temperature of a planet exposed to different levels of insolation. In both species, the response of the growth rate to local temperature is identical, but differences in albedo between the 2 species generate differences in local temperatures. The shifting balance between the daisies keeps the global temperature in a range suitable for life. Watson and Lovelock made the stronger claim that "the model always shows greater stability with daisies than it does without them." We examined this claim by introducing an extra source of competition into the equations that describe the interactions between the daisy species. Depending on the parameters of competition, temperatures can vary more widely with increasing insolation in the presence of daisies than without them. It now seems possible, timely and perhaps necessary, to include an accurate representation of interspecific competition when taking account of vegetational influences on climate.

1. Introduction

It has long been recognized that life influences physical and chemical environments and vice versa, at all scales from the local to the global (Lotka, 1925). Recent empirical and computational studies (Beerling et al., 1998) of Earth's *ménage à trois* — vegetation, climate and atmosphere — support that perspective.

One specific and controversial version of that widely accepted view is the Gaia hypothesis. In its early form (Kump, 1996), the Gaia hypothesis proposed that life affects the physical planet (including the climate and the chemical composition of the atmosphere and ocean) in ways that invariably or usually increase or maintain the suitability of Earth for life (Lovelock, 1988; Lenton, 1998). To demonstrate that planetary "homeostasis by and for the biosphere" could in principle work by mechanisms that entailed no teleology, Watson and Lovelock (1983) proposed a mathematical model called Daisyworld. They analyzed this model numerically. Saunders (1994) analyzed it mathematically.

Watson and Lovelock (1983) made a further strong claim for their "imaginary planet having ... just 2 species of daisy of different colors ... . Regardless of the details of the interaction, the effect of the daisies is to stabilize the temperature. ... the model always shows greater stability with daisies than it does without them." Lovelock (1988) later argued further for this claim but with fewer technical details.

Watson (1999, p. 83) articulated a more refined view of the Daisyworld model. He recognized

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that even one species of daisies diminishes changes in temperature in the midrange of insolation. However, life amplifies the temperature effect of small changes in insolation at the extremes where the daisies become just viable or just cease to be viable. "What is the essential difference between the behavior of the system with life and that without it? It is not increased stability, for though this is apparent in some regions, the opposite occurs in others. Rather, it is a change in the character of the system due to the non-linearity that is introduced by the equations governing the population of daisies."

This note describes a further example of the general statement of Watson (1999). We show that if interspecific competition affects daisy growth more than self-inhibition while all other assumptions of the Daisyworld model remain unchanged, then temperatures can vary more widely over some intervals of increasing insolation in the presence of the daisies than without them. The daisies need not stabilize the temperature of Daisyworld. This result suggests that global climatic dynamics may depend on the details of interactions within the biosphere.

This example is one of a growing number of variations on the theme of Daisyworld. Other variations are given by Maddock (1991), Saunders (1994), Harding and Lovelock (1996), Robertson and Robinson (1998) and the references cited by them and above.

2. Daisyworld with interspecific competition

In Daisyworld, a gray planet (with albedo $A_g = 0.5$) is seeded with black daisies (albedo $A_b = 0.25$) and white daisies (albedo $A_w = 0.75$). These 2 species have identical growth responses to local temperature. A sun delivers insolation that is fixed over time but is varied, in comparative statics, from 0.6 to 1.6× the insolation currently reaching earth. For each level of insolation, the Daisyworld model computes the planetary temperature using the Stefan–Boltzmann law for black body radiation and the average planetary albedo. The average albedo is determined by the fraction of planetary area α_b covered by black daisies, the fraction of planetary area α_w covered by white daisies, and the fraction of remaining area of bare gray ground $(P - \alpha_b - \alpha_w)$, where the potential daisy area P is always taken as 1 (by Watson and Lovelock and here). These areas are determined as the equilibrium of the growth equations of Carter and Prince (1981):

$$\frac{d\alpha_b}{dt} = \alpha_b((P - \alpha_b - \alpha_w)\beta(T_b) - \gamma),$$

$$\frac{d\alpha_w}{dt} = \alpha_w((P - \alpha_b - \alpha_w)\beta(T_w) - \gamma),$$

where γ is the death rate of daisies per unit time, and the growth function $\beta(T)$ (per unit time, per unit area) depends (identically for both species) on the local temperatures T_b, T_w in the areas of the planet covered by black and white daisies, respectively. Saunders (1994, p. 366) emphasizes that the patches of black and white daisies are spatially segregated, but a referee of this note argues that the model need not necessarily be interpreted as assuming spatial segregation because adjacent black and white objects may differ in temperature by several degrees. It would appear that the use of different local temperatures for regions of black and white daisies may require some spatial segregation, rather than random intermixing, of black and white daisies if the temperature differences between the black and white daisies are large enough. The growth function β is assumed to be 0 below 5°C, to rise in an inverted parabolic U to a value of 1 at 22.5°C, and to fall to 0 again at and above 40°C.

To represent interspecific competition we modified the above equations by assuming that

$$\frac{d\alpha_b}{dt} = \alpha_b((P - bb\alpha_b - wb\alpha_w)\beta(T_b) - \gamma_b),$$

$$\frac{d\alpha_w}{dt} = \alpha_w((P - bw\alpha_b - ww\alpha_w)\beta(T_w) - \gamma_w),$$

where bb is the density-dependent inhibitory effect of black daisies on black daisies, wb is the competitive effect of white daisies on black daisies, and so on. In our numerical explorations of the model, we held $bb = ww = 1$ as in the original model and varied only bw and wb. In all other respects, we used exactly the equations and parameter values of Watson and Lovelock (1983), with death rates per unit time $\gamma_b = \gamma_w = 0.3$.

3. Method of analysis

Watson and Lovelock solved their model by numerical iteration of its differential equations until the areas occupied by each species of daisy converged to an apparent steady-state. They compared different equilibria by repeated numerical solutions of the model with different parameters. By contrast, following the analytical approach of Saunders (1994), we assumed a steady-state by setting the right side of the above competition equations equal to 0. We then manipulated the resulting equations symbolically, together with the other equations of Watson and Lovelock, to obtain implicit equations for the state variables at equilibrium. We solved the implicit equations numerically and plotted the results. We carried out the symbolic manipulation, numerical solutions and plotting using Derive (Soft Warehouse, 1997). The reader who wishes the details of how we analyzed the model may obtain a technical description from either author.

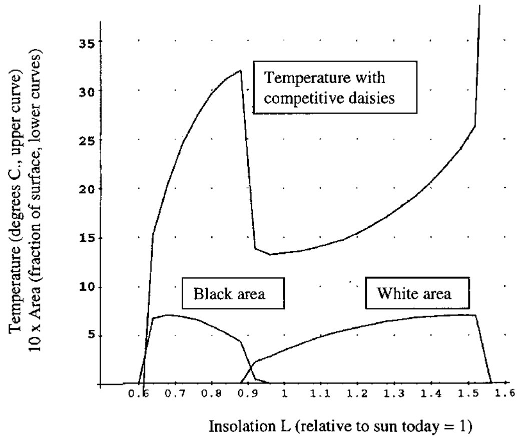

Fig. 1. Daisyworld temperature (T) as a function of insolation (L), expressed as a multiple of solar insolation, when density-dependent self-inhibition of each daisy species is considerably less than interspecific competition ($bb = ww = 1$, $bw = 2$, $wb = 3$). Between $L = 0.8$ and $L = 1.0$, the effective planetary temperature in the presence of daisies varies far more abruptly than it would in their absence or at lower levels of competition ($bw = wb = 1$). Daisy areas are scaled by $10\times$ their actual values (which always fall between 0 and 1).

4. Results

We first reproduced the prior results of Watson and Lovelock (1983). We omit our reproduction of their Fig. 1 because it has already been reproduced by several others. Although their Fig. 1 did not display a hysteresis at low luminosity, hysteresis does occur when a gradual decrease in luminosity is considered (Maddock, 1991, p. 332; Saunders, 1994, p. 369).

When competition between the black daisies and the white daisies is intense, a small change of insolation can drive a large change in steady-state temperature (Fig. 1). Many additional examples for other values of the competition parameters could also be given. When one of the interspecific competition coefficients retains its original value of 1 and the other interspecific competition coefficient is varied, the transition in temperature is generally less dramatic. The abruptness of the transition can also be modified by changing the parabolic dependence of plant growth on temperature to a Gaussian dependence (Harding and Lovelock, 1996, p. 110), as well as by many other changes in the model. These variations of the model modify the details of the effect of interspecific competition but do not change the qualitative conclusion that interspecific competition can amplify the effect on temperature of small changes in insolation.

When competition between daisy species is sufficiently intense, the abrupt temperature transition approximates a jump between the temperature trajectory of a world with black daisies only and the temperature trajectory of a world with white daisies only (T. M. Lenton, personal communication). As long as both kinds of daisies survive, the actual temperature trajectory is intermediate between these 2 extreme trajectories. An abrupt transition induced by interspecific competition does not drive the global temperature out of the range tolerable to life. Even with intense interspecific competition, Daisyworld is habitable over a wider range of insolation with daisies (of 1 species or 2) than it would be without them.

By numerical experimentation (not shown), we found that differences between the death rates of the black and the white daisies had a much smaller effect on the ruggedness of the temperature-insolation profile than did competition between the black and the white daisies.

5. Discussion: relevance to earth

The underlying supposition of Watson and Lovelock (1983) is that vegetation can modify global climate. Supporting that view, Hayden (1998) reviews many examples of the ways that vegetation modulates many aspects of climate at all scales, including the global scale. For example, vegetation increases the water vapor in the atmosphere through evapotranspiration. The increased moisture raises the minimum temperature and reduces the maximum temperature. Plants produce non-methane hydrocarbons, about half of which agglomerate as particulate hydrocarbons. At high relative humidities, water vapor condenses on particulate hydrocarbons to form haze and to raise minimum temperatures. The surface roughness of vegetation slows average wind speeds over the interior of England and Scotland by half, compared to wind speeds over the oceans adjacent to the UK. Different plants have different rates of evapotranspiration, produce different hydrocarbons, and have different forms of roughness and responses to wind velocity. Not all plants are interchangeable in their climatic effects.

Plant competition has long been recognized as significant in the composition of biotic communities (Clements et al., 1929; Grace and Tilman, 1990). Mathematical models have been developed to represent plant competition (Pakes and Maller, 1990). Maddock (1991, p. 336) considered interspecific competition in the context of the Daisyworld model, using a mathematical form that is slightly different from that used here, without exploring the extremes of competition considered here and without discussing possible mechanisms.

The interspecific competition modeled here could be produced by a well-known mechanism, allelopathy (Rice, 1984, 1995). Harper (1977, p. 369) wrote: "Some of the depressive effects of a plant upon its neighbours are so striking that an interpretation based on the monopolization of resources has often seemed inadequate. An alternative is obviously that some plants may release into their environment toxic materials that harm or even kill neighbours." Harper (1977) reviewed laboratory experiments that demonstrated the role of allelopathy. More recent studies (Rice, 1984, 1995) document the role of allelopathy in the field and support the assumption made here that a species may inhibit the growth of another species substantially more than it inhibits its own growth.

It seems possible, timely and perhaps necessary, to include an accurate representation of interspecific competition when taking account of vegetational influences on climate. Some general circulation models of the climate now include interaction with dynamic global vegetation models which allow for competition among functional types of vegetation (Foley et al., 1998; T. M. Lenton, personal communication). Such models are clearly a step in the right direction.

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The Effect of Competition on the Biological Homeostasis of Daisyworld

Andrew Watson and James Lovelock [1983] present equations describing a hypothetical planet they call Daisyworld. Two species of daisies grow on the planet – one dark colored (black) and one light colored (white). The authors aim to show that the plants will moderate the temperature to their own advantage despite changing amounts of solar luminosity.

...

To produce their results, Watson and Lovelock use a time simulation technique to compute the daisy areas and effective planet temperature at steady state for a given solar luminosity. We take a more analytical approach to the problem by determining the chain of equations that describes the model and then numerically solving these equations for a given luminosity. In addition, we include parameters in the equations to account for varying amounts of competition between the daisies. The following summarizes the procedure we use to model Daisyworld:

The growth rate β of either type of daisy is defined to be a parabolic function of the local temperature T_l between 5°C and ~~above~~ 40°C reaching a maximum of one at 22.5°C. The growth rate is zero below 5°C and above 40°C. This leads to the equation

$$\beta(T_l) = \max\left(0, 1 - \left(\frac{22.5 - T_l}{17.5}\right)^2\right) \tag{1}$$

The radiation absorbed by the planet is $S \cdot L \cdot (1 - A)$ where S is 9.17 10^2 watts/m^2, L is the solar luminosity, and A is the average albedo of the planet. The radiation emitted by the planet is $\sigma \cdot (Te + 273)^4$ where σ (Stefan's constant) is 5.67 10^{-8} watts/m^2/K^4 and Te is the effective temperature in °C at which the planet radiates. Since the absorbed and emitted radiation must be equal at steady state, this leads to the equation

$$S \cdot L \cdot (1 - A) = \sigma \cdot (Te + 273)^4 \tag{2}$$

Solving this equation for the albedo $A(L, Te)$ of the planet as a function of the solar luminosity and the effective temperature yields

$$A(L, Te) = 1 - \frac{\sigma}{S \cdot L} \cdot (Te + 273)^4 \tag{3}$$

The radiation per unit area emitted by a region of daisies is set equal to the radiation per unit area emitted by the planet as a whole plus a positive constant q times the difference between the average planet albedo A and the local albedo of the daisies A_l. This leads to the equation

$$(T_l + 273)^4 = (Te + 273)^4 + q \cdot (A - A_l) \tag{4}$$

Solving this equation for the local temperature $T_l(Te, A, A_l)$ as a function of the effective planet temperature and the planet and local albedos yields

$$T_l(Te, A, A_l) = \left((Te + 273)^4 + q \cdot (A - A_l)\right)^{\frac{1}{4}} - 273 \tag{5}$$

Watson and Lovelock use the comparative growth equations

$$\frac{d\alpha_b}{dt} = \alpha_b \cdot \left(\left(P - \alpha_b - \alpha_w \right) \cdot \beta\left(T_b \right) - \gamma \right)$$
$$\frac{d\alpha_w}{dt} = \alpha_w \cdot \left(\left(P - \alpha_b - \alpha_w \right) \cdot \beta\left(T_w \right) - \gamma \right) \tag{6}$$

due to Carter and Prince [1981] to describe the rate of change of the areas α of black and white daisies versus time. P is the fraction of the planet's area capable of growing daisies, $\beta\left(T_i\right)$ is the growth rate per unit time per unit area, and γ is the death rate per unit time.

To account for varying levels of competition between the daisies and different black and white daisy death rates we generalize these simultaneous growth equations to

$$\frac{d\alpha_b}{dt} = \alpha_b \cdot \left(\left(P - bb \cdot \alpha_b - wb \cdot \alpha_w \right) \cdot \beta\left(T_b \right) - \gamma_b \right)$$
$$\frac{d\alpha_w}{dt} = \alpha_w \cdot \left(\left(P - bb \cdot \alpha_b - wb \cdot \alpha_w \right) \cdot \beta\left(T_w \right) - \gamma_w \right) \tag{7}$$

where γ_b is the black death rate per unit time, γ_w is the white death rate per unit time, bb is the inhibitory effect of black daisies on black daisies, wb is the inhibitory effect of white daisies on black daisies, etc.

If at steady state only black daisies are surviving (i.e. $\alpha_w = 0$), then the following equation must be satisfied

$$\left(P - bb \cdot \alpha_b \right) \cdot \beta\left(T_b \right) - \gamma_b = 0 \tag{8}$$

Solving this equation for the black daisy area $\alpha_b\left(T_b\right)$ as a function of its local temperature yields

$$\alpha_b\left(T_b\right) = \frac{P - \dfrac{\gamma_b}{\beta\left(T_b\right)}}{bb} \tag{9}$$

Similarly, if at steady state only white daisies are surviving (i.e. $\alpha_b = 0$), then the white daisy area $\alpha_w\left(T_w\right)$ as a function of its local temperature is

$$\alpha_w\left(T_w\right) = \frac{P - \dfrac{\gamma_w}{\beta\left(T_w\right)}}{ww} \tag{10}$$

If at steady state both black and white daisies are surviving (i.e. their areas are nonzero), then by equation set (7) the following equations must be satisfied

$$bb \cdot \alpha_b + wb \cdot \alpha_w = P - \frac{\gamma_b}{\beta\left(T_b\right)}$$
$$bw \cdot \alpha_b + ww \cdot \alpha_w = P - \frac{\gamma_w}{\beta\left(T_w\right)} \tag{11}$$

Solving this simultaneous system of linear equations for the black and white daisy areas $\alpha_b\left(T_b, T_w\right)$ and $\alpha_w\left(T_b, T_w\right)$ as functions of local temperatures yields

$$\alpha_b(T_b, T_w) = \frac{ww \cdot \left(P - \frac{\gamma_b}{\beta(T_b)} \right) - wb \cdot \left(P - \frac{\gamma_w}{\beta(T_w)} \right)}{bb \cdot ww - bw \cdot wb}$$

$$\alpha_w(T_b, T_w) = \frac{bw \cdot \left(P - \frac{\gamma_b}{\beta(T_b)} \right) - bb \cdot \left(P - \frac{\gamma_w}{\beta(T_w)} \right)}{bw \cdot wb - bb \cdot ww} \tag{12}$$

provided that the system is nonsingular (*i.e.* $bb \cdot ww \neq bw \cdot wb$). However, if the system is singular, it must be the case that

$$bw \cdot \left(P - \frac{\gamma_b}{\beta(T_b)} \right) = bb \cdot \left(P - \frac{\gamma_w}{\beta(T_w)} \right) \tag{13}$$

provided that both black and white daisies are surviving.

The average albedo of the planet can be found by summing the products of the albedos of the various regions on the planet times their respective areas. This ~~yields~~ leads to the equation

$$A = \alpha_g \cdot A_g + \alpha_b \cdot A_b + \alpha_w \cdot A_w \tag{14}$$

Since the area of bare ground α_g equals $1 - \alpha_b - \alpha_w$, equation (14) can be rewritten as

$$A = A_g + \alpha_b \cdot \left(A_b - A_g \right) + \alpha_w \cdot \left(A_w - A_g \right) \tag{15}$$

Now we have all the equations required to determine the planet's effective temperature Te for a given solar luminosity L. First, we consider the case when the system of equations (11) is nonsingular:

If both black and white daisies are surviving, equation set (12) defines the areas α_b and α_w as functions of the local temperatures T_b and T_w. Equation (5) defines the local temperatures T_b and T_w as functions of the effective temperature Te and planet albedo A. Equation (3) defines the planet albedo A as a function of solar luminosity L and effective temperature Te. In summary

$$\alpha_b = \alpha_b\Big(\beta\big(T_b(Te, A(L, Te))\big), \beta\big(T_w(Te, A(L, Te))\big)\Big)$$

$$\alpha_w = \alpha_w\Big(\beta\big(T_b(Te, A(L, Te))\big), \beta\big(T_w(Te, A(L, Te))\big)\Big) \tag{16}$$

and

$$A = A(L, Te)$$

If only black daisies are surviving, the situation is much simpler since by equation (9)

$$\alpha_b = \alpha_b\Big(\beta\big(T_b(Te, A(L, Te))\big)\Big)$$

$$\alpha_w = 0 \tag{17}$$

and

$$A = A(L, Te)$$

Similarly if only white daisies are surviving, by equation (10)

$$\alpha_b = 0$$

$$\alpha_w = \alpha_w\Big(\beta\big(T_w(Te, A(L, Te))\big)\Big) \tag{18}$$

and

$$A = A(L, Te)$$

Substituting the expressions for α_b, α_w, and A in equation set (16) into equation (15) yields an equation that only depends on the solar luminosity L, the effective temperature Te, and various constant parameters. The same is true if the expressions for α_b, α_w, and A in equation sets (17) or (18) are substituted into equation (15). Therefore, in all three cases the effective temperature is an implicitly defined function of the solar luminosity.

Now the question becomes: Which of the three equations is the appropriate one to use for a given luminosity? Our solution is to assume that both black and white daisies are surviving by substituting equation set (16) into equation (15) and numerically solving for the effective temperature. Then given this effective temperature, use equation set (16) to compute the black and white daisy areas and see if they are, in fact, positive. If so, the assumption is valid, and the effective temperature and daisy areas have been computed.

If the white daisy area is nonpositive, assume that only black daisies are surviving by substituting equation set (17) into equation (15) and numerically solving for the effective temperature. Then given this effective temperature, use equation set (17) to compute the black daisy area and see if it is, in fact, positive. If so, the assumption is valid, and the effective temperature and daisy areas have been computed; if not, the planet is lifeless, and the albedo of the planet A equals the albedo of bare ground A_g. Thus, the effective temperature of a lifeless planet $Te_g(L)$ as a function of solar luminosity can be derived from equation (3) to be

$$Te_g(L) = \left(\frac{S \cdot L \cdot (1 - A_g)}{\sigma} \right)^{\frac{1}{4}} - 273 \tag{19}$$

Similarly, if the black daisy area is nonpositive, assume that only white daisies are surviving by using equation set (18) to compute the effective temperature and verifying that the white daisy area is positive. If the white daisy area is not positive, the planet is lifeless and equation (19) can again be used to compute the effective temperature.

The above procedure for computing the effective temperature and daisy areas for a given solar luminosity assumed that the system of equations (11) is nonsingular. Now, we consider the case when the system is singular:

Equation (5) defines the local temperatures T_b and T_w as functions of the effective temperature Te and planet albedo A. Equation (3) defines the planet albedo A as a function of solar luminosity L and effective temperature Te. In summary

$$\begin{aligned} T_b &= T_b(Te, A(L, Te)) \\ T_w &= T_w(Te, A(L, Te)) \end{aligned} \tag{20}$$

Substituting the expressions for T_b and T_w in equation set (20) into equation (13) yields an equation that depends only on the solar luminosity L, the effective temperature Te, and various constant parameters. Therefore, the effective temperature is an implicitly defined function of the solar luminosity. Note that this equation is valid only if both black and white daisies are surviving, since equation (13) depends on this assumption.

As with the nonsingular case, we begin by assuming that both black and white daisies are surviving by substituting equation set (20) into equation (13) and numerically solving for the effective temperature. Then given this effective temperature, we need to compute the black and white daisy areas and see if they are, in fact, positive. Unfortunately, equation set (20) does not provide formulas for computing daisy areas. However, equations (11) and (15) can be combined to make the system of equations

$$
\begin{aligned}
bb \cdot \alpha_b + wb \cdot \alpha_w &= P - \frac{\gamma_b}{\beta(T_b)} \\
(A_b - A_g) \cdot \alpha_b + (A_w - A_g) \cdot \alpha_w &= A - A_g
\end{aligned}
\tag{21}
$$

Solving this system of linear equations for the black and white daisy areas $\alpha_b(T_b, A)$ and $\alpha_w(T_b, A)$ as functions of local black temperature and the planet albedo yields

$$
\begin{aligned}
\alpha_b(T_b, A) &= \frac{\left(P - \dfrac{\gamma_b}{\beta(T_b)}\right) \cdot (A_g - A_w) + wb \cdot (A - A_g)}{A_g \cdot (bb - wb) + A_b \cdot wb - A_w \cdot bb} \\
\alpha_w(T_b, A) &= \frac{\left(P - \dfrac{\gamma_b}{\beta(T_b)}\right) \cdot (A_b - A_g) - bb \cdot (A - A_g)}{A_g \cdot (bb - wb) + A_b \cdot wb - A_w \cdot bb}
\end{aligned}
\tag{22}
$$

Using equations (1), (3), (5), and (22), the black and white daisy areas can be computed as a function of the solar luminosity and effective temperature. Now the same procedure used for the nonsingular case can be used to verify the assumption that both black and white daisies are surviving. If not, the fact that the system of equations (11) is singular is of no consequence. Thus, the same procedure described for the nonsingular case can be used to compute the effective temperature and daisy areas, if either black or white daisies are not surviving.

Watson and Lovelock use the following values for the various Daisyworld parameters:

$A_g = 0.5$ the albedo of bare ground not covered daisies

$A_b = 0.25$ the albedo of ground covered by black daisies

$A_w = 0.75$ the albedo of ground covered by white daisies

$P = 1$ the fraction planet area that is fertile

$\gamma_b = \gamma_w = 0.3$ the death rate per unit time

$q = 4 \cdot (273 + 22.5)^3 \cdot 20 = 2.06425 \cdot 10^9$ the insulation factor

Their model is the special case when $bb = bw = wb = ww = 1$.

Notes for Article

1. Local temperatures imply color segregation
2. Thermal radiation balance equation W&L 1983 (4) implies steady state wrt biotic changes.
3. Linear approximation to W&L 1983 (6) not used.
4. Note that one of multiple solutions to implicitly defined "functions" is taken.
5. When both daisies are surviving in a singular daisy world (i.e. bb · ww = bw · wb), the local temperatures are constant wrt luminosity.

$$E = \sigma T^4$$

$$\sigma = 1.36 \times 10^{-4} \text{ kcal m}^{-2} \text{ s}^{-1} \text{ K}^{-4}$$

$$= 5.67 \times 10^{-8} \text{ watt m}^{-2} \text{ K}^{-4}$$

$$= 5.67 \times 10^{-5} \text{ erg cm}^{-2} \text{ sec}^{-1} \text{ K}^{-4}$$

Condon & Odishaw

5-32 $\sigma = 5.6694 \times 10^{-5} \frac{erg}{cm^2 sec\, deg^4} = 5.67 \times$

6-15 $\sigma = 5.672 \times 10^{-5} \frac{erg}{cm^2 deg^4}$

$$\left(\frac{T}{645}\right)^4 = \text{total radiant flux in } \frac{watts}{cm^2}$$

$$645^{-4} = 5.78 \times 10^{-12}$$

$1 \text{ watt} = 10^7 \text{ erg/sec}$

$$5.67 \times 10^{-5} \frac{watts}{m^2 K^4} = 5.67 \times 10^{-5} \cdot \frac{10^7 erg}{10^4 sec\, \cancel{10}\, cm^2 K^4} = 5.67 \times 10^{-2} \frac{erg}{sec\, cm^2 K^4}$$

This agrees with Pennycuick p. 35

Figures

b bb bw (ww) wb

baseline: W+L parameters D1111 with
all white
all black } 1-daisy model
no daisies

Interesting

D1011 , D1012

D1110 D1201

D1,1.5,1,1.5

D1212

γ_w: .2 .3 .4

γ_b: .2 .3 .4

[InputMode := Word, CaseMode := Sensitive, PrecisionDigits := 8]

"Physical constants:"

$\sigma := 5.67\cdot 10^{-5}$ Stefan's constant

$S := 9.17\cdot 10^{5}$ Flux constant

"Daisyworld equations:"

$q := 4\cdot(273 + 22.5)^{3}\cdot \mathrm{qprime}$ Solar energy redistribution constant

$A(L, Te) := 1 - \frac{\sigma}{S\cdot L}\cdot(Te + 273)^{4}$ Planet albedo

$Tb(L, Te) := \mathrm{qprime}\cdot(A(L, Te) - Ab) + Te$ Black daisy local temperature approximation

$Tw(L, Te) := \mathrm{qprime}\cdot(A(L, Te) - Aw) + Te$ White daisy local temperature approximation

$Tb(L, Te) := (q\cdot(A(L, Te) - Ab) + (Te + 273)^{4})^{1/4} - 273$ Black daisy local temperature

$Tw(L, Te) := (q\cdot(A(L, Te) - Aw) + (Te + 273)^{4})^{1/4} - 273$ White daisy local temperature

$\beta b(L, Te) := 1 - \left(\frac{22.5 - Tb(L, Te)}{17.5}\right)^{2}$ Black daisy growth rate

$\beta w(L, Te) := 1 - \left(\frac{22.5 - Tw(L, Te)}{17.5}\right)^{2}$ White daisy growth rate

Black daisy area w/ zero determinant

$$\alpha b0(L, Te) := \frac{\left(P - \frac{\gamma b}{\beta b(L, Te)}\right)\cdot(Ag - Aw) + wb\cdot(A(L, Te) - Ag)}{Ag\cdot(bb - wb) + Ab\cdot wb - Aw\cdot bb}$$

White daisy area w/ zero determinant

$$\alpha w0(L, Te) := \frac{\left(P - \frac{\gamma b}{\beta b(L, Te)}\right)\cdot(Ab - Ag) - bb\cdot(A(L, Te) - Ag)}{Ag\cdot(bb - wb) + Ab\cdot wb - Aw\cdot bb}$$

Black daisy area (nonsingular)

$$\alpha b(L, Te) := \mathrm{MAX}\left(0,\ \mathrm{IF}\left(\beta b(L, Te) \le 0 \vee \beta w(L, Te) \le 0,\ ?,\ \frac{ww\cdot\left(P - \frac{\gamma b}{\beta b(L, Te)}\right) - wb\cdot\left(P - \frac{\gamma w}{\beta w(L, Te)}\right)}{bb\cdot ww - bw\cdot wb}\right)\right)$$

White daisy area (nonsingular)

$$\alpha w(L, Te) := MAX\left(0, IF\left(\beta w(L, Te) \le 0 \vee \beta b(L, Te) \le 0, ?, \frac{bw\cdot\left(P - \frac{\gamma b}{\beta b(L, Te)}\right) - bb\cdot\left(P - \frac{\gamma w}{\beta w(L, Te)}\right)}{bw\cdot wb - bb\cdot ww}\right)\right)$$

Converts a data matrix to matrices of pairs

$$PairOff(matrix) := VECTOR(VECTOR(\left[row_1, row_i\right], row, matrix), i, 2, DIMENSION(matrix_1))$$

Extend DaisyWorld data matrix

$$FixUp(data) := VECTOR(\left[r_1, r_2, 10\cdot r_3, 10\cdot r_4, IF(r_3 > 0, Tb(r_1, r_2), ?), IF(r_4 > 0, Tw(r_1, r_2), ?), IF(r_3 > 0, 10\cdot\beta b(r_1, r_2), ?), IF(r_4 > 0, 10\cdot\beta w(r_1, r_2), ?)\right], r, data)$$

$$BlackOnlyAux(L, T, b) := IF\left(b < 0 \vee T = 0, \left[L, \left(\frac{S\cdot L\cdot(1 - Ag)}{\sigma}\right)^{1/4} - 273, 0, 0\right], [L, T, b, 0]\right)$$

$$BlackOnly(L, Te) := BlackOnlyAux\left(L, Te, \frac{A(L, Te) - Ag}{Ab - Ag}\right)$$

$$WhiteOnlyAux(L, T, w) := IF\left(w < 0 \vee T = 0, \left[L, \left(\frac{S\cdot L\cdot(1 - Ag)}{\sigma}\right)^{1/4} - 273, 0, 0\right], [L, T, 0, w]\right)$$

$$WhiteOnly(L, Te) := WhiteOnlyAux\left(L, Te, \frac{A(L, Te) - Ag}{Aw - Ag}\right)$$

$$\text{BlackAndOrWhite}(L, T, b, w) := \text{IF}\left(b \le 0,\ \text{IF}\left(w \le 0,\ [L, T, 0, 0],\ \text{WhiteOnly}\left(L,\ \text{RHS}\left(\left(\text{SOLVE}\left(A(L, Te) = Ag + \frac{P - \dfrac{\gamma w}{\text{MAX}(0,\ \beta w(L, Te))}}{ww}\cdot(Aw - Ag),\ Te,\ -20,\ 32\right)\right)_1\right)\right)\right),\ \text{IF}\left(w \le 0,\ \text{BlackOnly}\left(L,\ \text{RHS}\left(\left(\text{SOLVE}\left(A(L, Te) = Ag + \frac{P - \dfrac{\gamma b}{\text{MAX}(0,\ \beta b(L, Te))}}{bb}\cdot(Ab - Ag),\ Te,\ 8,\ 80\right)\right)_1\right)\right),\ [L, T, b, w]\right)\right)$$

$$\text{DaisyZeroDet}(L, Te) := \text{BlackAndOrWhite}(L, Te, \alpha b0(L, Te), \alpha w0(L, Te))$$

$$\text{DaisyNonzeroDet}(L, Te) := \text{BlackAndOrWhite}(L, Te, \alpha b(L, Te), \alpha w(L, Te))$$

~~User~~

$$\text{DaisyData}(lower, upper, step) := \text{FixUp}\left(\text{VECTOR}\left(\text{IF}\left(bb\cdot ww = bw\cdot wb,\ \text{DaisyZeroDet}\left(L,\ \text{RHS}\left(\left(\text{SOLVE}\left(bw\cdot\left(P - \frac{\gamma b}{\text{MAX}(0,\ \beta b(L, Te))}\right) = bb\cdot\left(P - \frac{\gamma w}{\text{MAX}(0,\ \beta w(L, Te))}\right),\ Te,\ 10,\ 60\right)\right)_1\right)\right),\ \text{DaisyNonzeroDet}(L,\ \text{RHS}((\text{SOLVE}(A(L, Te) = Ag + \alpha b(L, Te)\cdot(Ab - Ag) + \alpha w(L, Te)\cdot(Aw - Ag),\ Te,\ -20,\ 80)))_1))\right),\ L,\ lower,\ upper,\ step\right)\right)$$

Generate plot matrix
DaisyPlot(lower, upper, step) := PairOff(DaisyData(lower, upper, step))

"Daisyworld parameters:"

$[Ag := 0.5,\ Ab := 0.25,\ Aw := 0.75,\ An := 0.5]$ Daisy albedos

$[\gamma b := 0.3,\ \gamma w := 0.3]$ Daisy death rates

$P := 1$ Fraction of planet's area that is fertile ground

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qprime := 20                                  q' - Solar energy redistribution constant

[bb := 1, wb := 1, bw := 1, ww := 1]

DaisyPlot(0.56, 1.64, 0.02)                   To generate plot data matrix
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```
; File: DAISY.LSP                        07/09/97

(FLAG '"compete" 'FUNCTION)
(DEFUN "compete" (ARG1 ARG2 ARG3 ARG4)
  ((AND (NUMBERP ARG1) (NUMBERP ARG2) (NUMBERP ARG3) (NUMBERP ARG4))
    (SETQ BB ARG1  BW ARG2  WW ARG3  WB ARG4)
    (SETQ DESCRIM (- (* BB WW) (* BW WB))) )
  (MAKE-VECTOR (LIST (== "bb" BB) (== "bw" BW) (== "ww" WW) (== "wb" WB))) )

(FLAG '"albedo" 'FUNCTION)
(DEFUN "albedo" (ARG1 ARG2 ARG3 ARG4)
  ((AND (NUMBERP ARG1) (NUMBERP ARG2) (NUMBERP ARG3) (NUMBERP ARG4))
    (SETQ AG ARG1  AB ARG2  AW ARG3  AN ARG4) )
  (MAKE-VECTOR (LIST (== "àg" AG) (== "àb" AB) (== "àw" AW) (== "àn" AN))) )

(FLAG '"misc" 'FUNCTION)
(DEFUN "misc" (ARG1 ARG2 ARG3 ARG4)
  ((AND (NUMBERP ARG1) (NUMBERP ARG2) (NUMBERP ARG3) (NUMBERP ARG4))
    (SETQ GAMMAB ARG1  GAMMAW ARG2  QPRIME ARG3  P ARG4)
    (SETQ Q (* 4 (^ (+ 273 22.5) 3) QPRIME)) )
  (MAKE-VECTOR (LIST (== "gammab" GAMMAB) (== "gammaw" GAMMAW)
                     (== "qprime" QPRIME) (== "P" P))) )

(FLAG '"descrim" 'FUNCTION)
(DEFUN "descrim" (ARG) DESCRIM)


(FLAG 'A 'FUNCTION)
(DEFUN A (L TE)
  (ALBEDO L TE) )

(FLAG '"Tw" 'FUNCTION)
(DEFUN "Tw" (L TE)
  (TL TE (ALBEDO L TE) AW) )

(FLAG '"Tb" 'FUNCTION)
(DEFUN "Tb" (L TE)
  (TL TE (ALBEDO L TE) AB) )

(FLAG '"Teg" 'FUNCTION)
(DEFUN "Teg" (L)
  (- (^ (/ (* L (- 1 AG)) SS) 1/4) 273) )


(FLAG '"àb0" 'FUNCTION)
(DEFUN "àb0" (L TE          ; Black daisy area on a black/white world
   AE)
  ((AND (NUMBERP L) (NUMBERP TE))
    (SETQ AE (ALBEDO L TE))
    (/ (+ (* (- P (/ GAMMAB ("beta" (TL TE AE AB)))) (- AG AW))
          (* (- AE AG) WB))
       (+ (* (- BB WB) AG) (* AB WB) (* AW BB -1))) )
  (LIST '"àb0" L TE) )
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```
(FLAG '"àw0" 'FUNCTION)
(DEFUN "àw0" (L TE          ; White daisy area on a black/white world
    AE)
  ((AND (NUMBERP L) (NUMBERP TE))
    (SETQ AE (ALBEDO L TE))
    (/ (- (* (- P (/ GAMMAB ("beta" (TL TE AE AB)))) (- AB AG))
          (* (- AE AG) BB))
       (+ (* (- BB WB) AG) (* AB WB) (* AW BB -1))) )
  (LIST '"àw0" L TE) )

(FLAG '"àb" 'FUNCTION)
(DEFUN "àb" (L TE)          ; Black daisy area on a black/white world
  ((AND (NUMBERP L) (NUMBERP TE))
    (CADR (BLACKWHITE-AREAS L TE)) )
  (LIST '"àb" L TE) )

(FLAG '"àw" 'FUNCTION)
(DEFUN "àw" (L TE)          ; White daisy area on a black/white world
  ((AND (NUMBERP L) (NUMBERP TE))
    (CADDR (BLACKWHITE-AREAS L TE)) )
  (LIST '"àw" L TE) )


(FLAG '"BlackOnly" 'FUNCTION)
(DEFUN "BlackOnly" (L TE
    AE AREA-B)
  (SETQ AE (ALBEDO L TE)
        AREA-B (/ (- AE AG) (- AB AG)))
  ((OR (<= AREA-B 0) (= TE 0))
    (MAKE-VECTOR (LIST L ("Teg" L) 0 0)) )
  (MAKE-VECTOR (LIST L TE AREA-B 0)) )

(FLAG '"WhiteOnly" 'FUNCTION)
(DEFUN "WhiteOnly" (L TE
    AE AREA-W)
  (SETQ AE (ALBEDO L TE)
        AREA-W (/ (- AE AG) (- AW AG)))
  ((OR (<= AREA-W 0) (= TE 0))
    (MAKE-VECTOR (LIST L ("Teg" L) 0 0)) )
  (MAKE-VECTOR (LIST L TE 0 AREA-W)) )


(FLAG '"BlackEquation" 'FUNCTION)
(DEFUN "BlackEquation" (L TE
    PAIR)
  ((AND (NUMBERP L) (NUMBERP TE))
    (SETQ PAIR (BLACK-AREA L TE))
    (- (CAR PAIR) AG (* (CADR PAIR) (- AB AG))) )
  (LIST '"BlackEquation" L TE) )

(FLAG '"WhiteEquation" 'FUNCTION)
(DEFUN "WhiteEquation" (L TE
    PAIR)
  ((AND (NUMBERP L) (NUMBERP TE))
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    (SETQ PAIR (WHITE-AREA L TE))
    (- (CAR PAIR) AG (* (CADR PAIR) (- AW AG))) )
  (LIST '"WhiteEquation" L TE) )


(FLAG '"BlackWhiteEquationZeroDet" 'FUNCTION)
(DEFUN "BlackWhiteEquationZeroDet" (L TE
    AE)
  ((AND (NUMBERP L) (NUMBERP TE))
    (SETQ AE (ALBEDO L TE))
    (- (* BW (- P (/ GAMMAB (MAX 0 ("beta" (TL TE AE AB))))))
       (* BB (- P (/ GAMMAW (MAX 0 ("beta" (TL TE AE AW))))))) )
  (LIST ' "BlackWhiteEquationZeroDet" L TE) )

(FLAG '"BlackWhiteEquation" 'FUNCTION)
(DEFUN "BlackWhiteEquation" (L TE
    PAIR)
  ((AND (NUMBERP L) (NUMBERP TE))
    (SETQ PAIR (BLACKWHITE-AREAS L TE))
    (- (CAR PAIR) AG (* (CADR PAIR) (- AB AG)) (* (CADDR PAIR) (- AW AG))) )
  (LIST ' "BlackWhiteEquation" L TE) )


(DEFUN BLACK-AREA (L TE
; Returns a list of planet albedo and black daisy area.
    AE)
  (SETQ AE (ALBEDO L TE))
  (LIST AE (/ (- P (/ GAMMAB (MAX 0 ("beta" (TL TE AE AB))))) BB)) )

(DEFUN WHITE-AREA (L TE
; Returns a list of planet albedo and white daisy area.
    AE)
  (SETQ AE (ALBEDO L TE))
  (LIST AE (/ (- P (/ GAMMAW (MAX 0 ("beta" (TL TE AE AW))))) WW)) )

(DEFUN BLACKWHITE-AREAS (L TE
; Returns a list of planet albedo, black daisy area, and white daisy area
; assuming both black and white daisies are surviving.
    AE BETAB BETAW DELTAB DELTAW AREA-B AREA-W)
  (SETQ AE (ALBEDO L TE)
        BETAB ("beta" (TL TE AE AB))
        BETAW ("beta" (TL TE AE AW))
        DELTAB (- P (/ GAMMAB BETAB))
        DELTAW (- P (/ GAMMAW BETAW)))
  ( ((OR (<= BETAB 0) (<= BETAW 0))
      (SETQ AREA-B *?*
            AREA-W *?*) )
    (SETQ AREA-B (/ (- (* WW DELTAB) (* WB DELTAW)) DESCRIM)
          AREA-W (/ (- (* BB DELTAW) (* BW DELTAB)) DESCRIM)) )
  (LIST AE AREA-B AREA-W) )

(FLAG '"beta" 'FUNCTION)
(DEFUN "beta" (TL)                          ; Growth rate equation
  (- 1 (^ (/ (- 22.5 TL) 17.5) 2)) )
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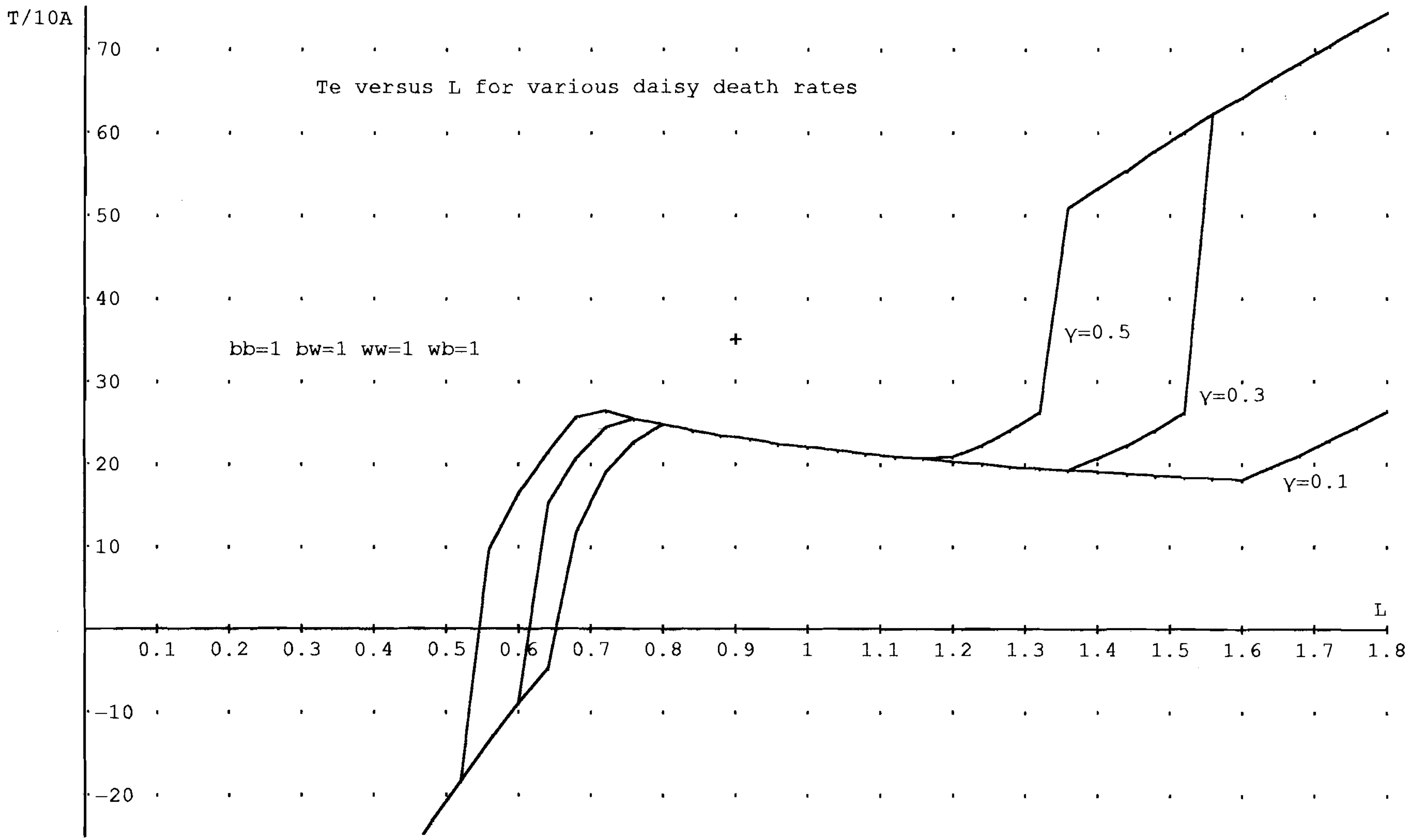
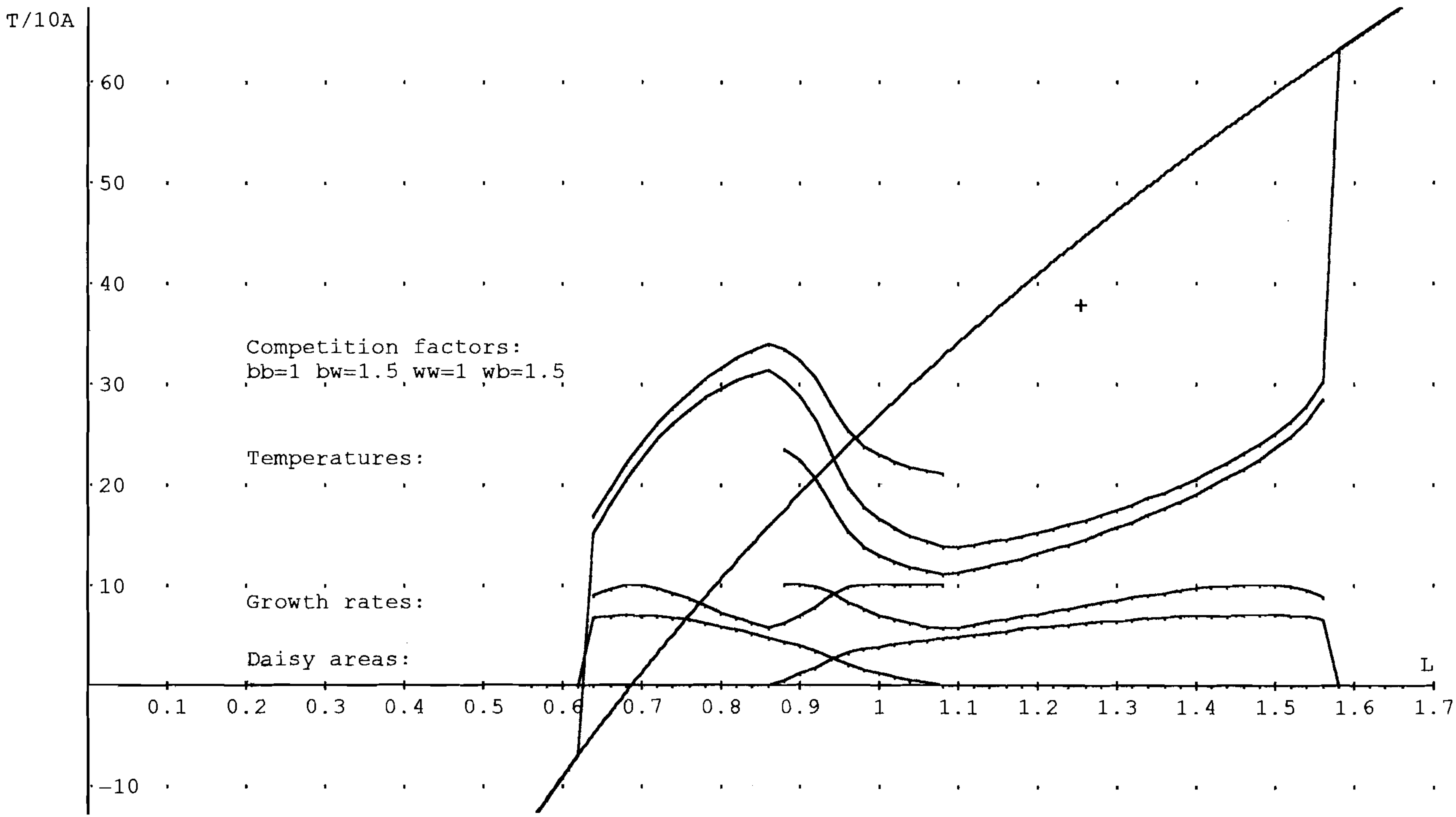
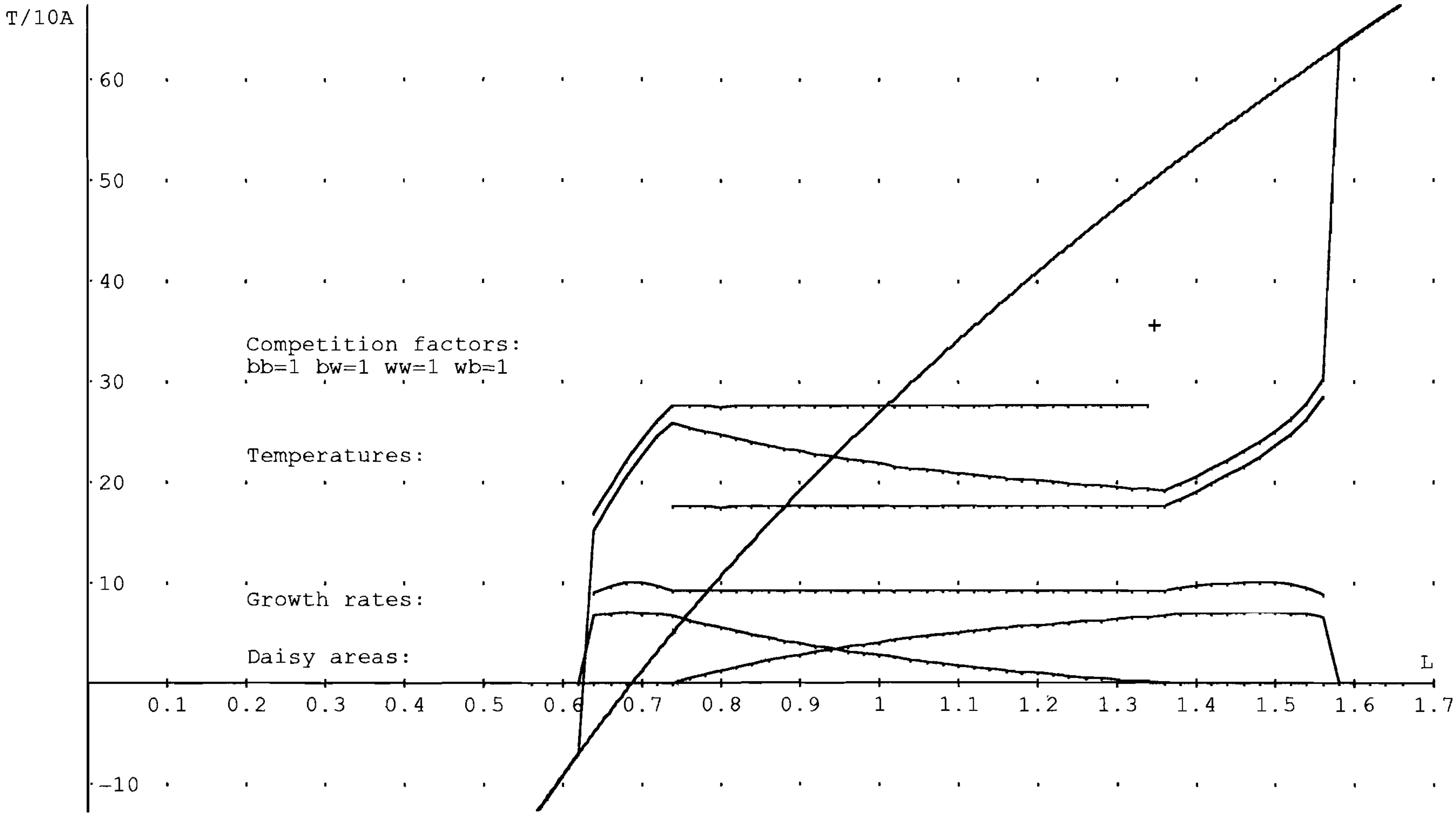
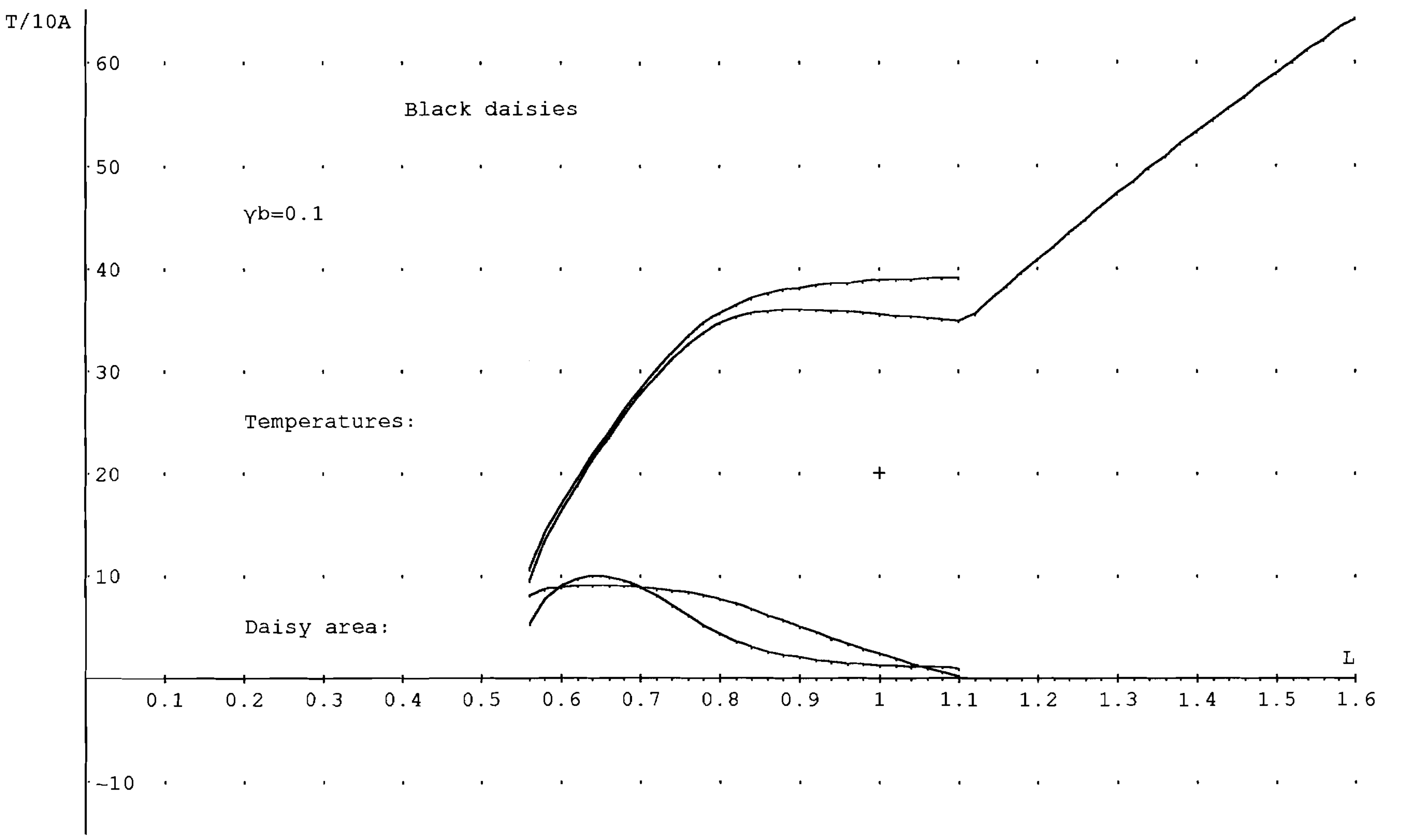
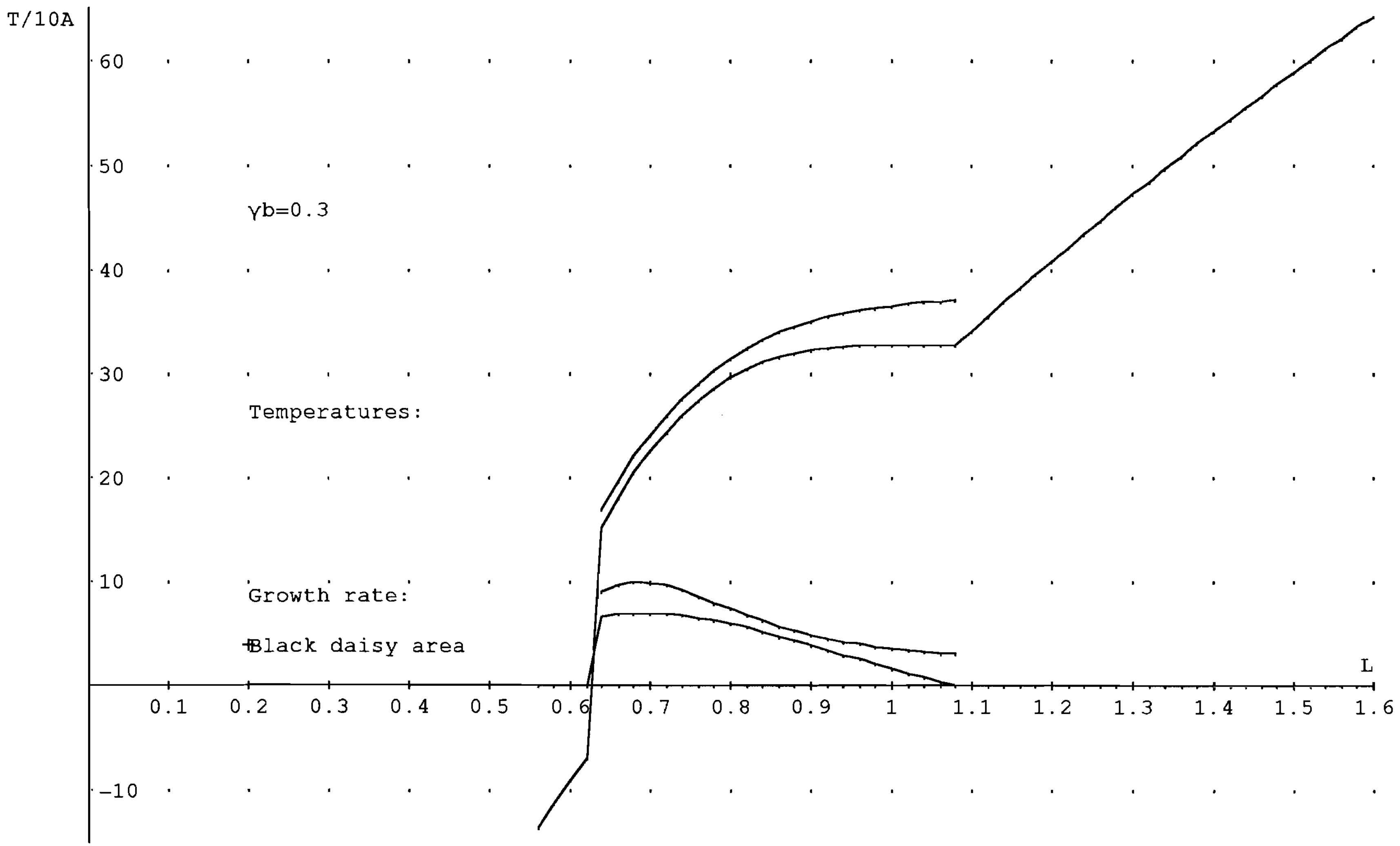
```
(DEFUN ALBEDO (L TE)                        ; Planet albedo
  (- 1 (/ (* SS (^ (+ TE 273) 4)) L)) )

(DEFUN TL (TE AE AL)                        ; Local temperature
  (- (^ (+ (* Q (- AE AL)) (^ (+ TE 273) 4)) 1/4) 273) )

(DEFUN TL (TE AE AL)                        ; Local temperature
  (+ (* QPRIME (- AE AL)) TE) )

(SETQ SS (/ 0.0000567 917000))

("compete" 1 1 1 1)
("albedo" 0.5 0.25 0.75 0.5)
("misc" 0.3 0.3 20 1)
```

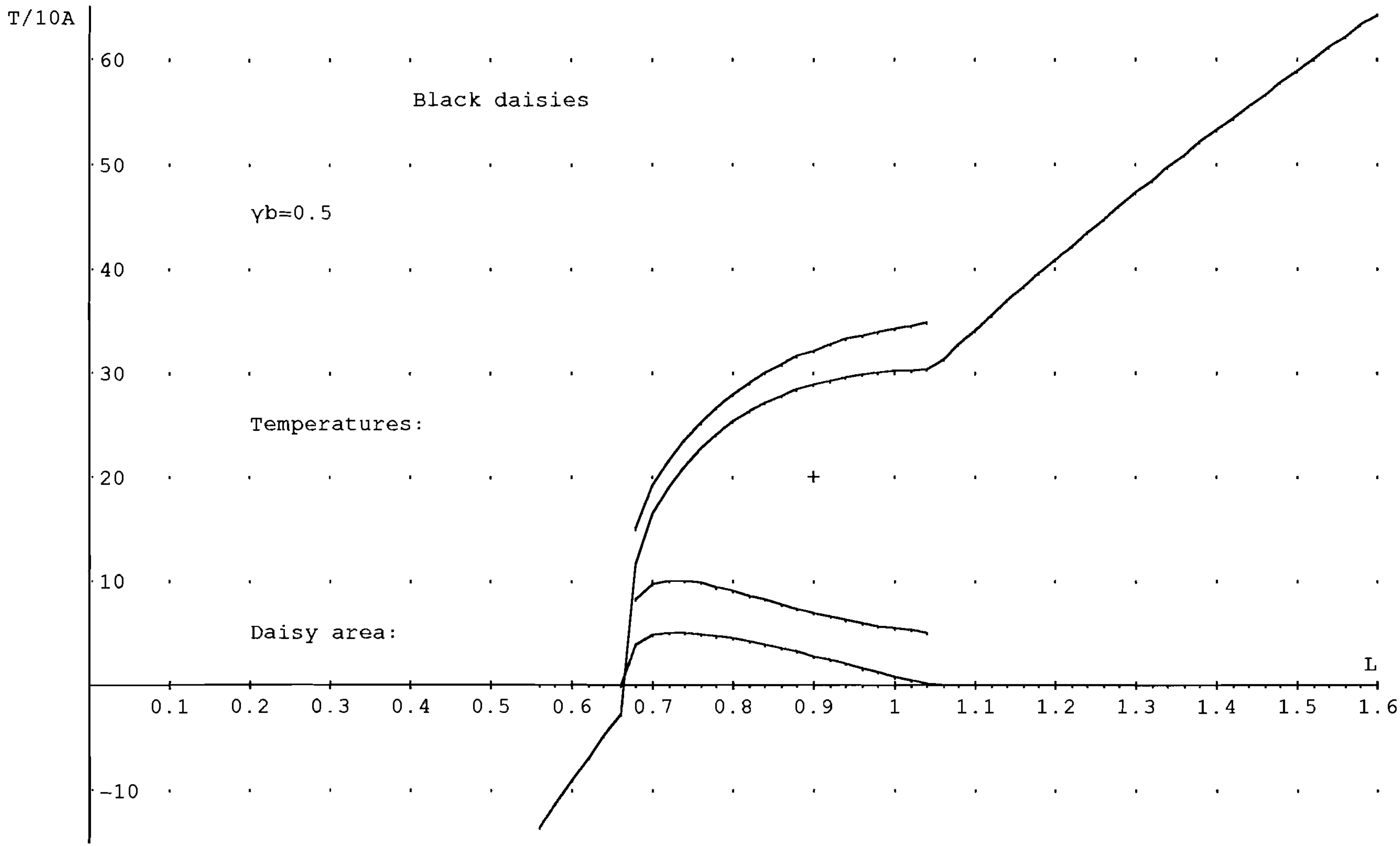

T/10A
Te versus L for various daisy death rates
bb=1 bw=1 ww=1 wb=1
γ=0.5
γ=0.3
γ=0.1
L
70
60
50
40
30
20
10
−10
−20
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8

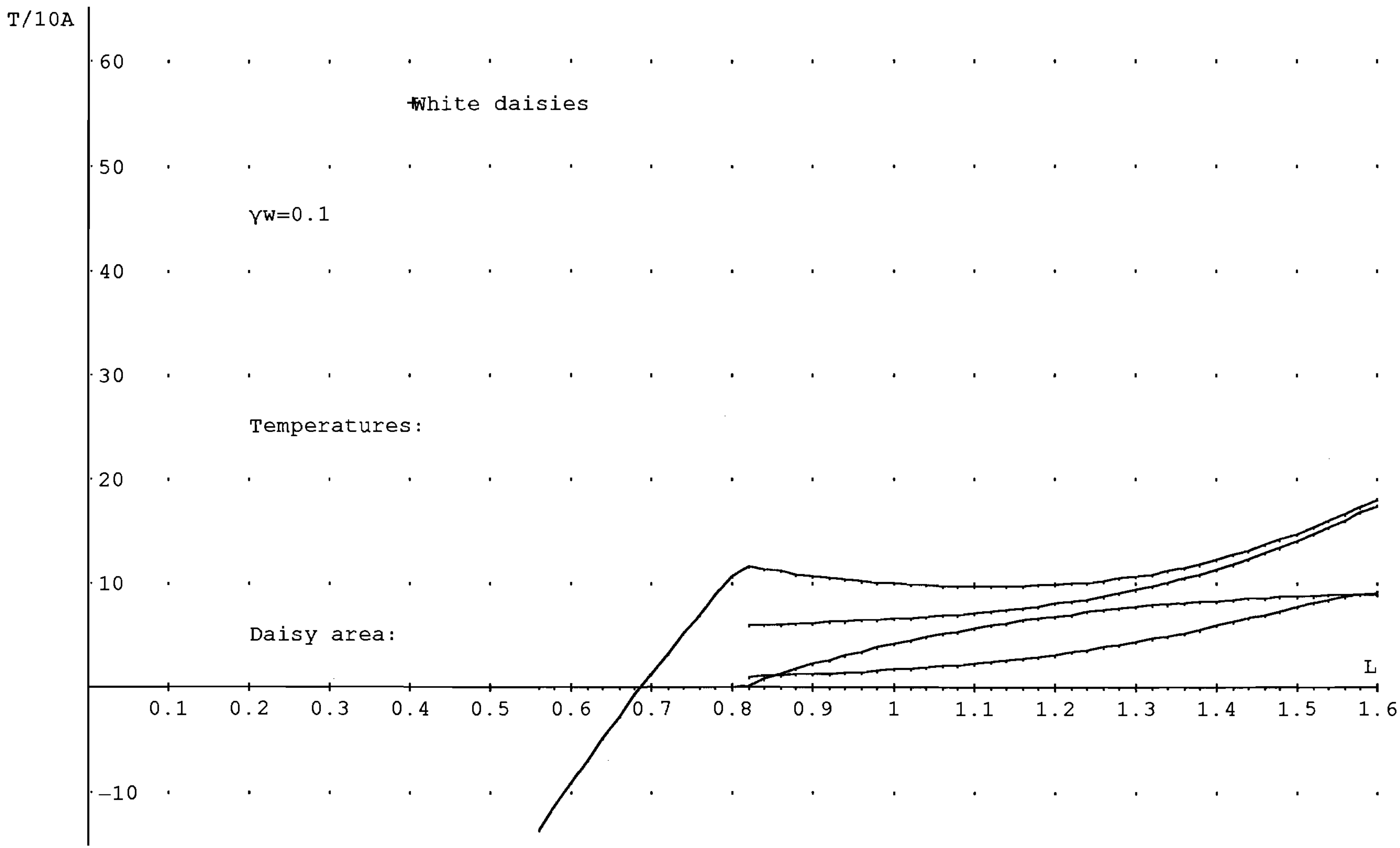

T/10A
60
50
40
30
20
10
-10
Competition factors:
bb=1 bw=1.5 ww=1 wb=1.5
Temperatures:
Growth rates:
Daisy areas:
L
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1 1.1 1.2 1.3 1.4 1.5 1.6 1.7

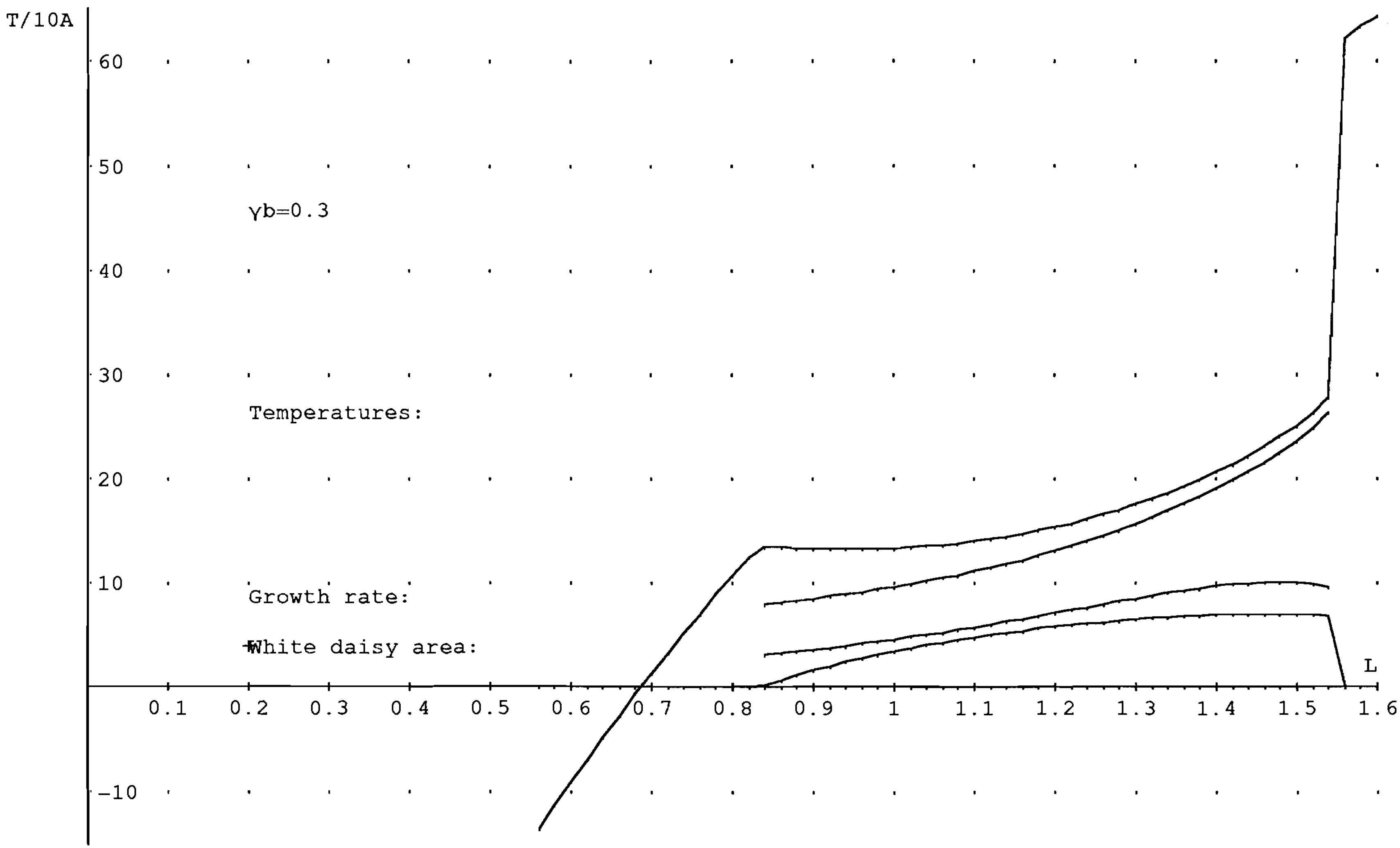

T/10A
60
50
40
30
20
10
-10
Competition factors:
bb=1 bw=1 ww=1 wb=1
Temperatures:
Growth rates:
Daisy areas:
L
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
1.1
1.2
1.3
1.4
1.5
1.6
1.7

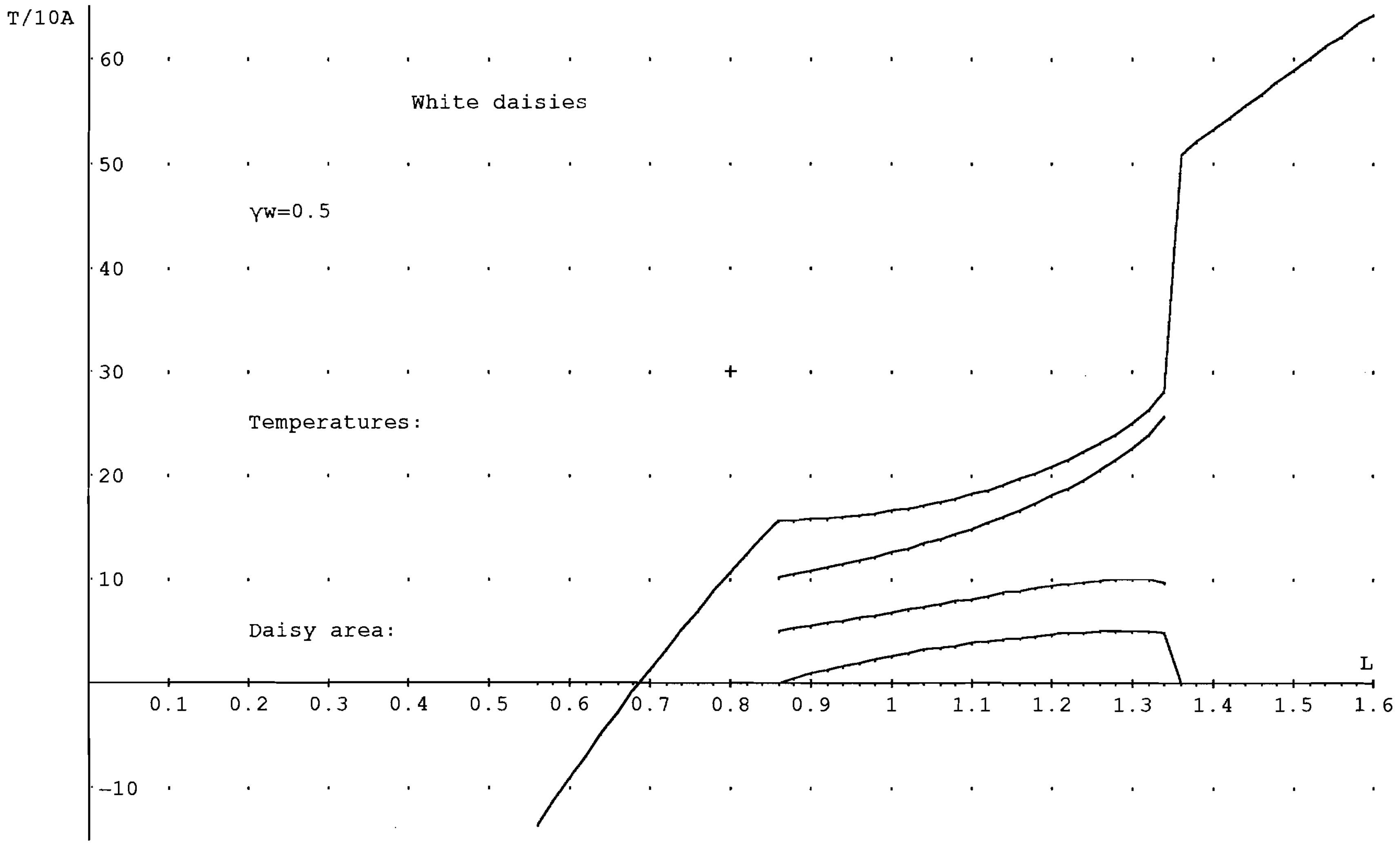

T/10A
60
50
40
30
20
10
-10
Black daisies
γb=0.1
Temperatures:
Daisy area:
L
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1 1.1 1.2 1.3 1.4 1.5 1.6

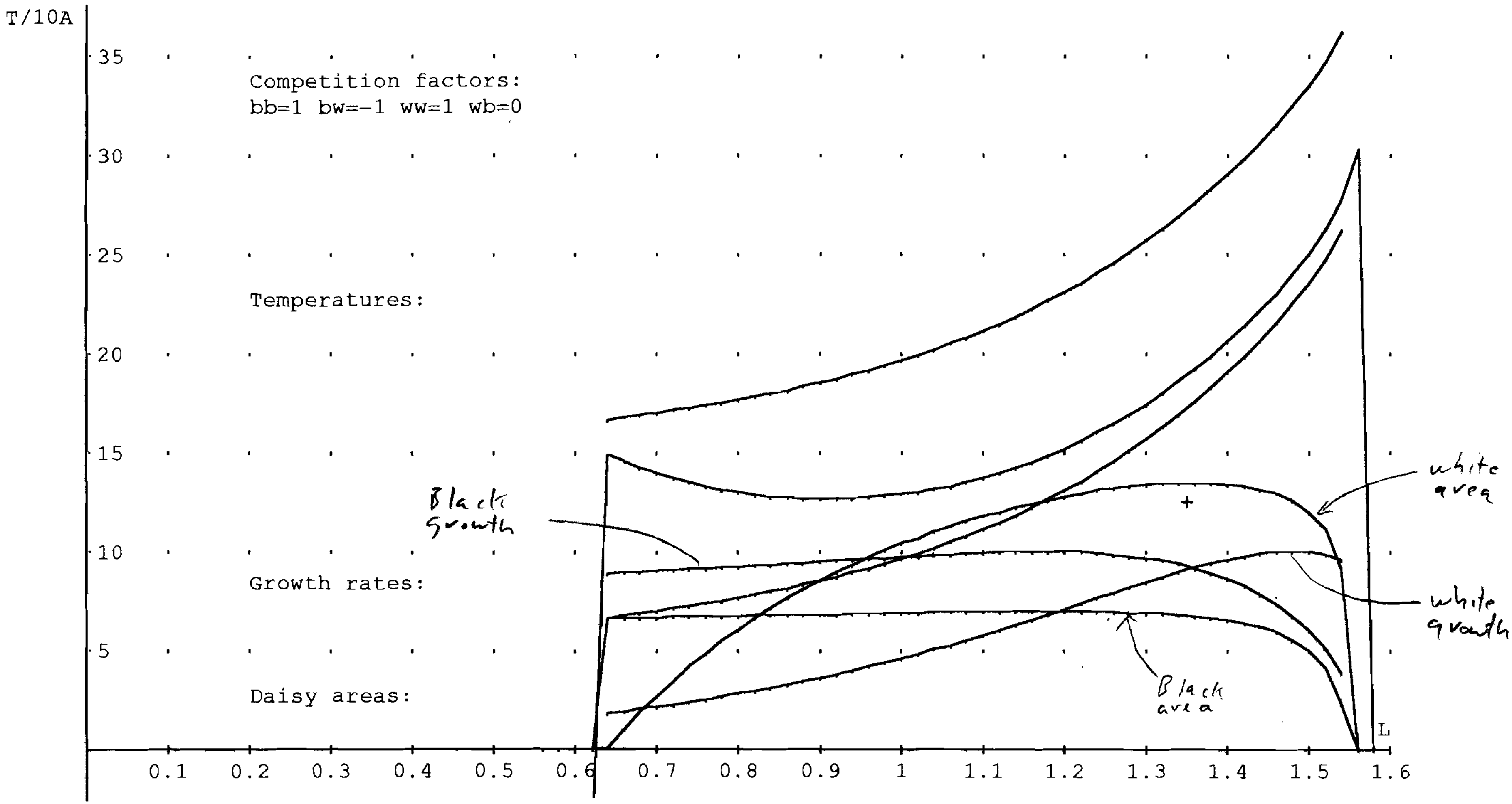

T/10A
60
50
40
30
20
10
-10
γb=0.3
Temperatures:
Growth rate:
Black daisy area
L
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
1.1
1.2
1.3
1.4
1.5
1.6

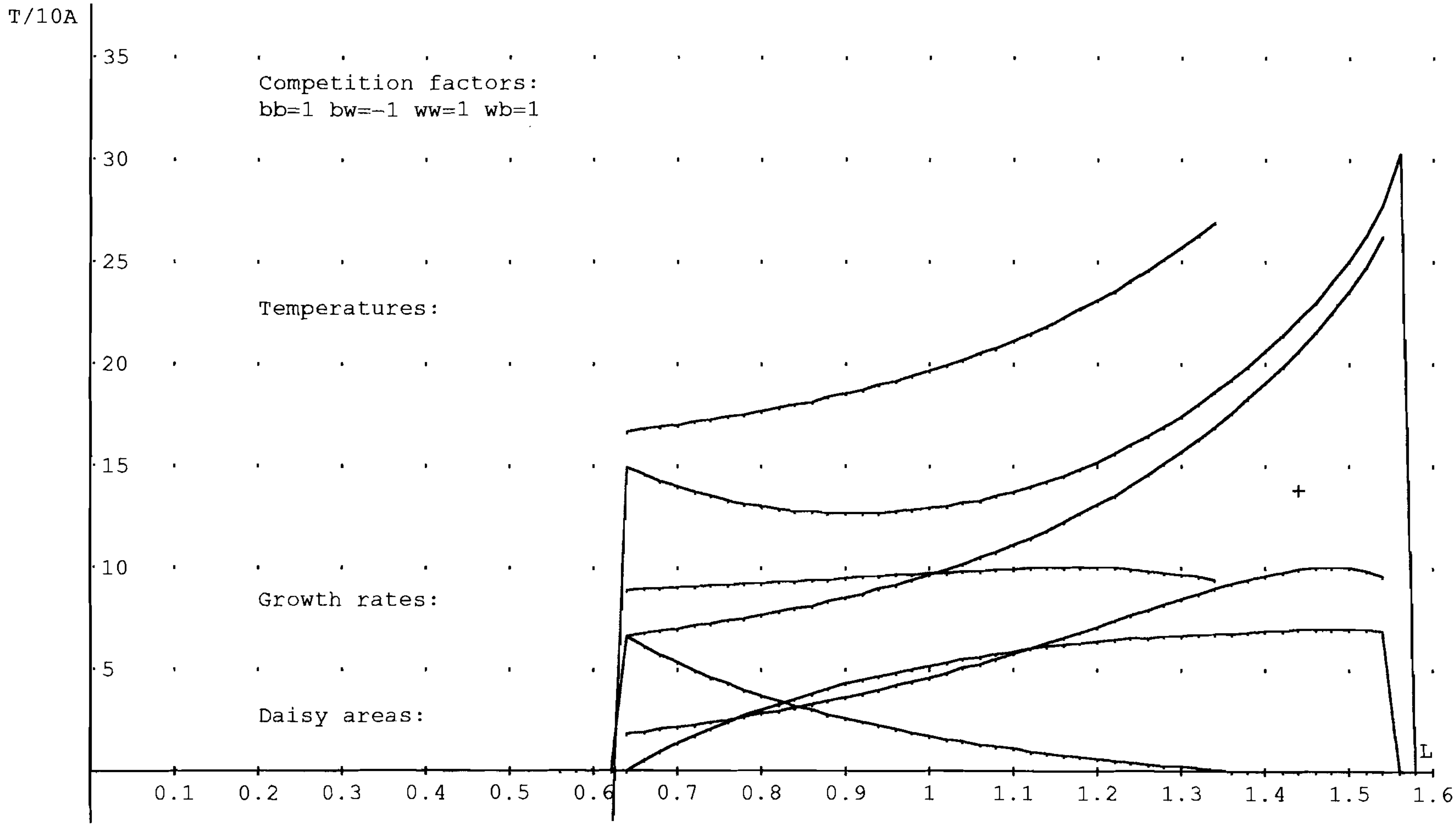

T/10A
60
50
40
30
20
10
-10
Black daisies
γb=0.5
Temperatures:
Daisy area:
L
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
1.1
1.2
1.3
1.4
1.5
1.6

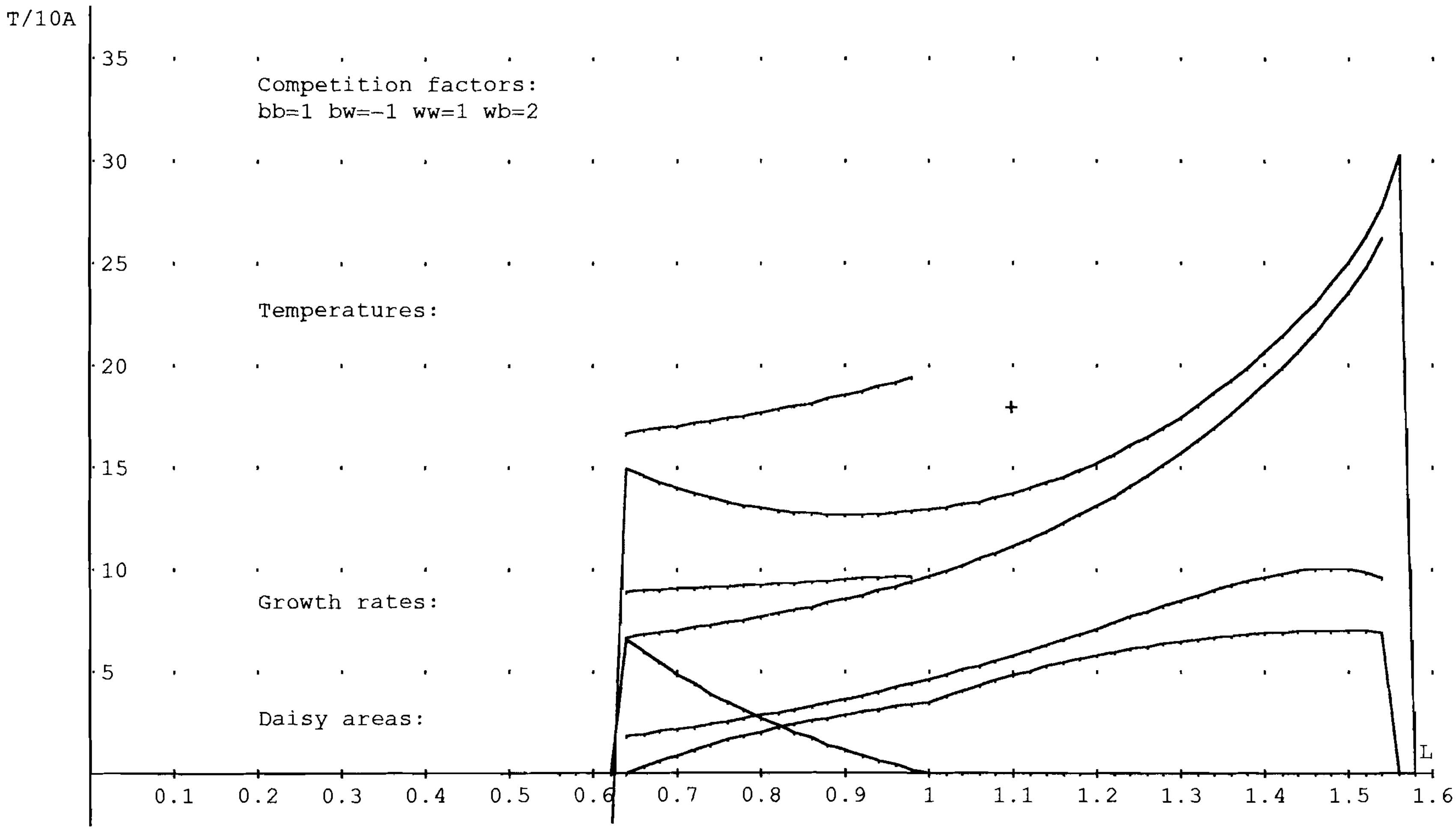

T/10A
60
White daisies
50
γw=0.1
40
30
Temperatures:
20
10
Daisy area:
L
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1 1.1 1.2 1.3 1.4 1.5 1.6
−10

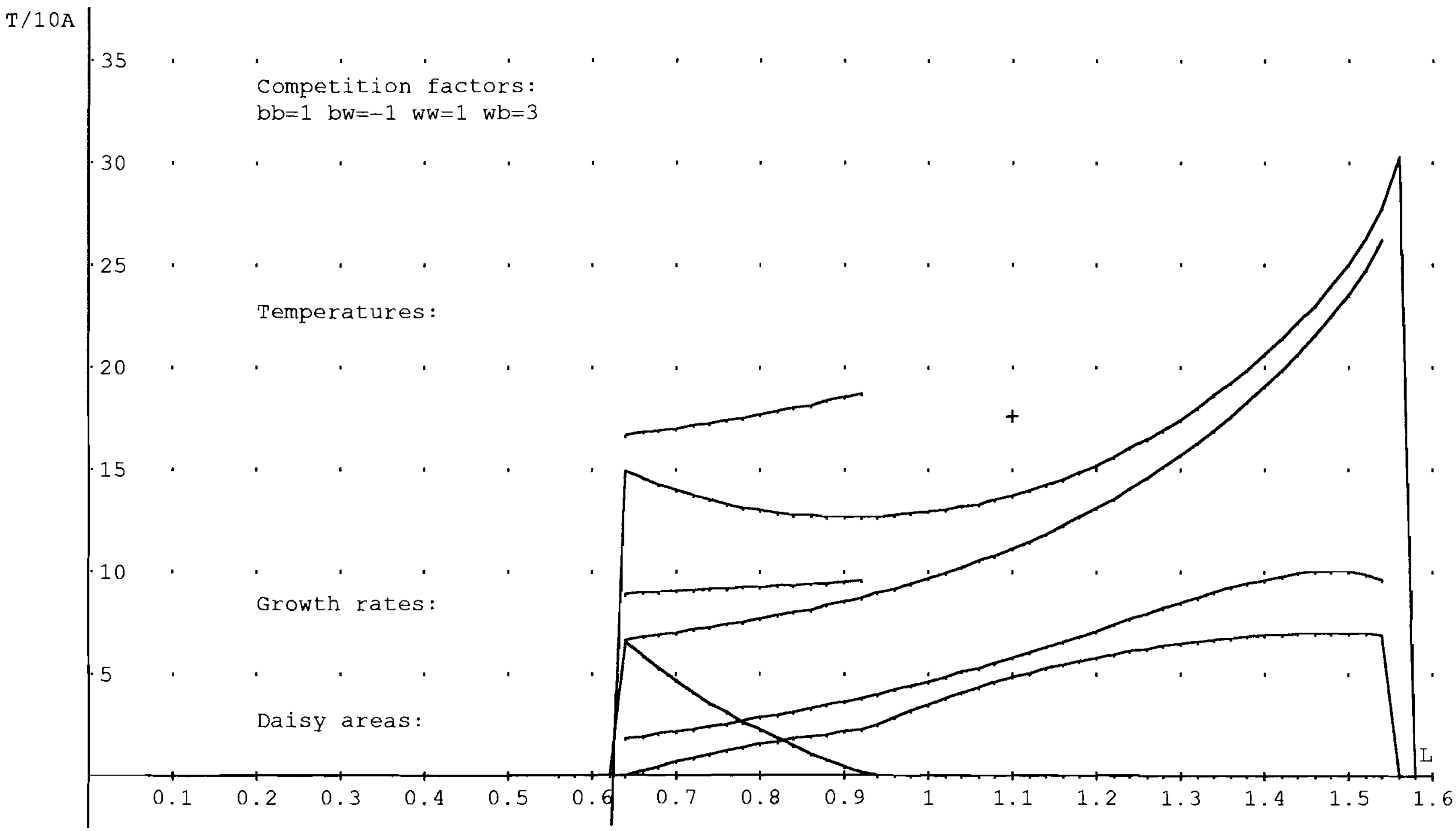

T/10A
60
50
γb=0.3
40
30
Temperatures:
20
10
Growth rate:
White daisy area:
L
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
1.1
1.2
1.3
1.4
1.5
1.6
-10


T/10A
60
50
40
30
20
10
-10
White daisies
γw=0.5
Temperatures:
Daisy area:
L
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
1.1
1.2
1.3
1.4
1.5
1.6


T/10A
35
30
25
20
15
10
5
Competition factors:
bb=1 bw=-1 ww=1 wb=0
Temperatures:
Growth rates:
Daisy areas:
Black growth
white area
white growth
Black area
L
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
1.1
1.2
1.3
1.4
1.5
1.6


T/10A
35
30
25
20
15
10
5
Competition factors:
bb=1 bw=-1 ww=1 wb=1
Temperatures:
Growth rates:
Daisy areas:
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
1.1
1.2
1.3
1.4
1.5
1.6
L


T/10A
35
30
25
20
15
10
5
Competition factors:
bb=1 bw=-1 ww=1 wb=2
Temperatures:
Growth rates:
Daisy areas:
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
1.1
1.2
1.3
1.4
1.5
1.6
L


T/10A
35
30
25
20
15
10
5
Competition factors:
bb=1 bw=-1 ww=1 wb=3
Temperatures:
Growth rates:
Daisy areas:
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
1.1
1.2
1.3
1.4
1.5
1.6
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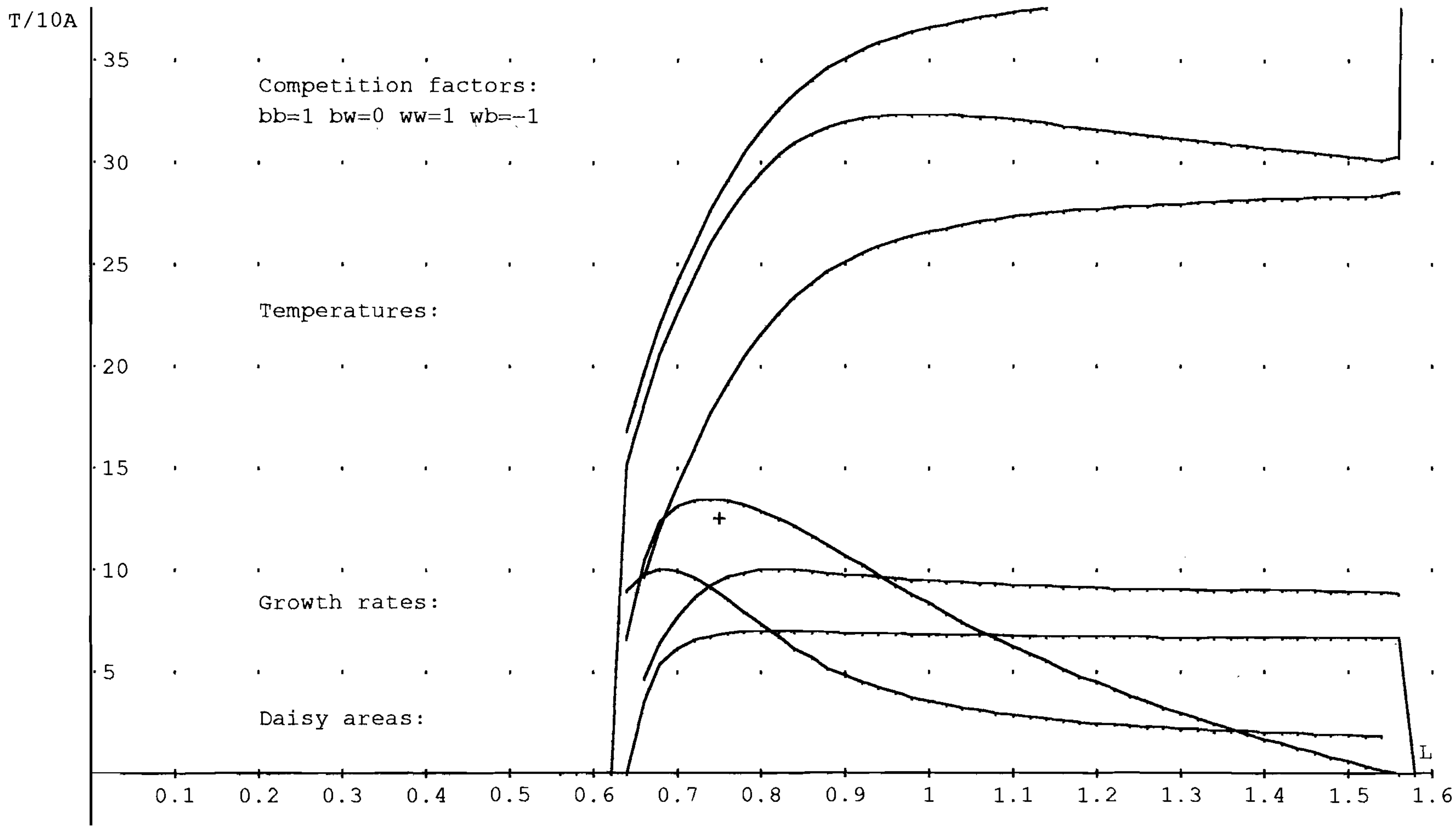
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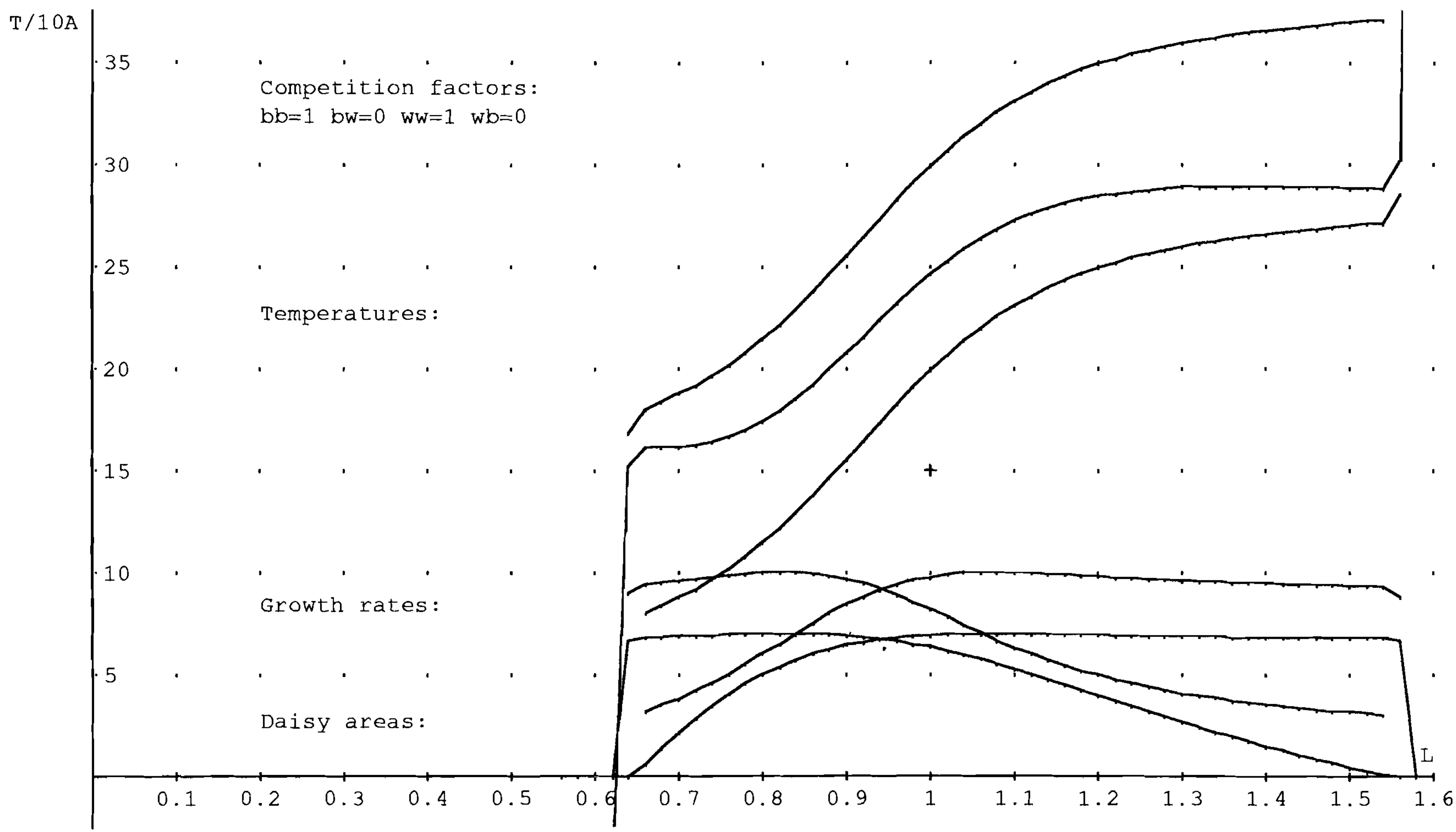

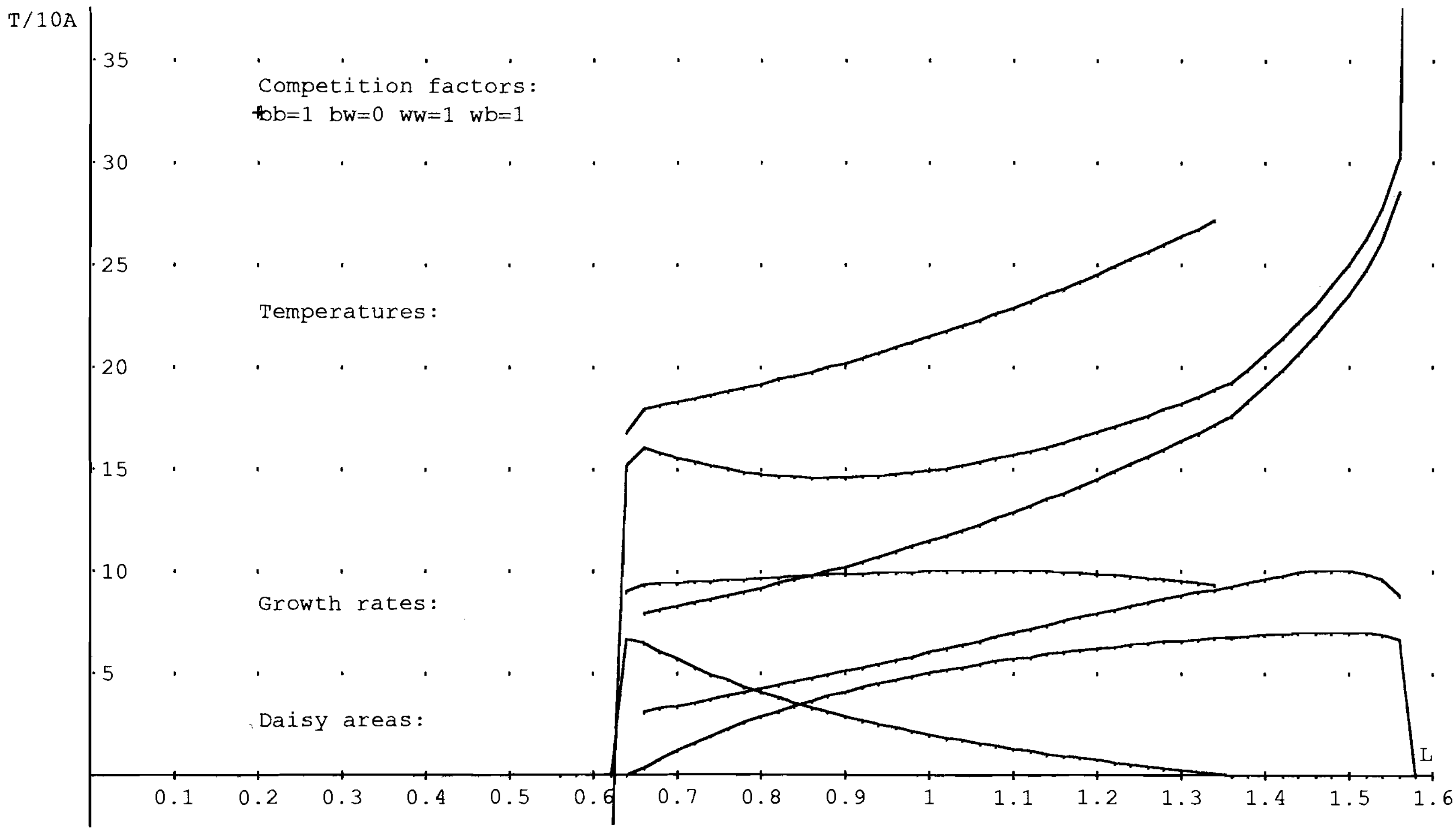
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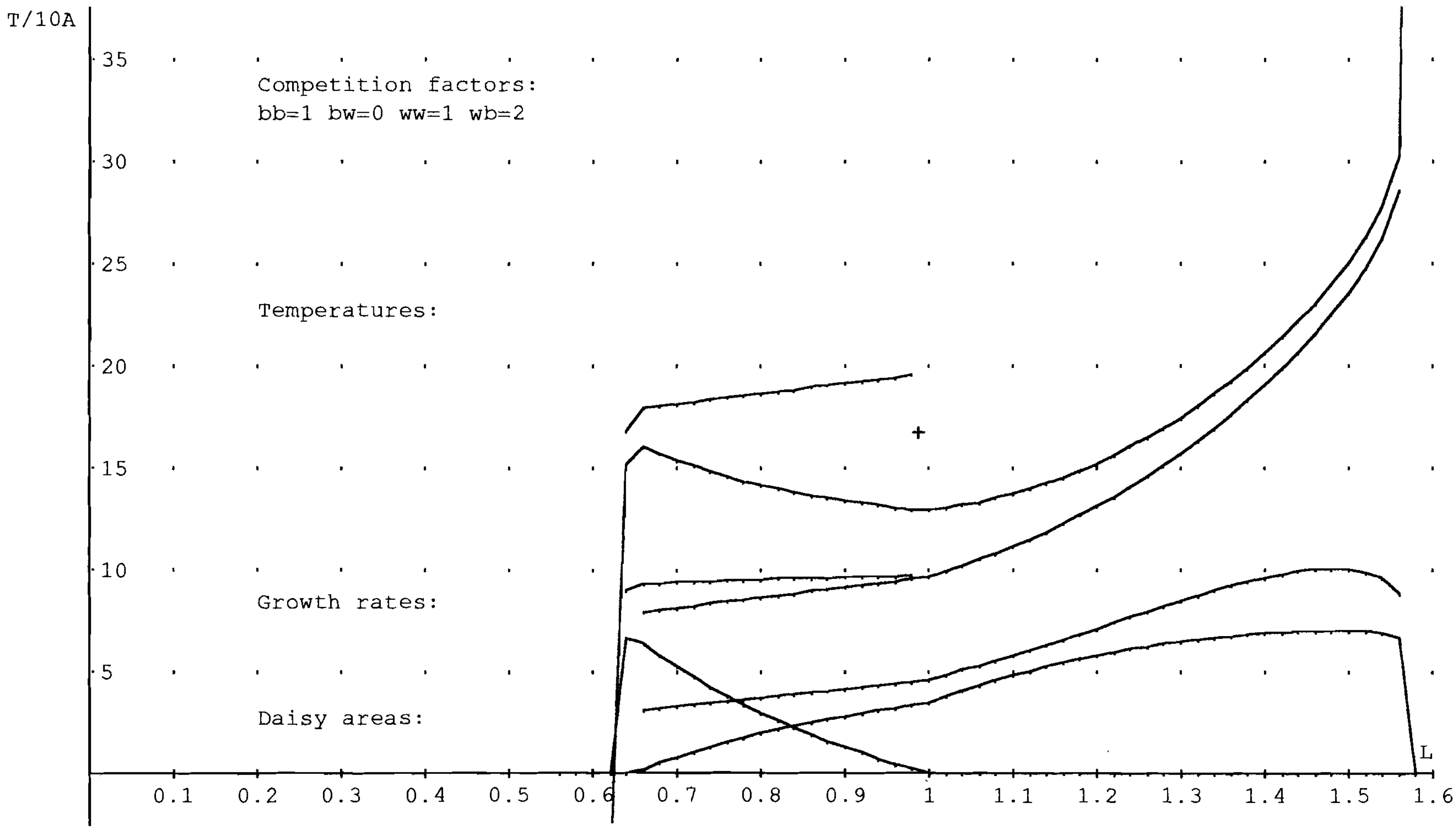
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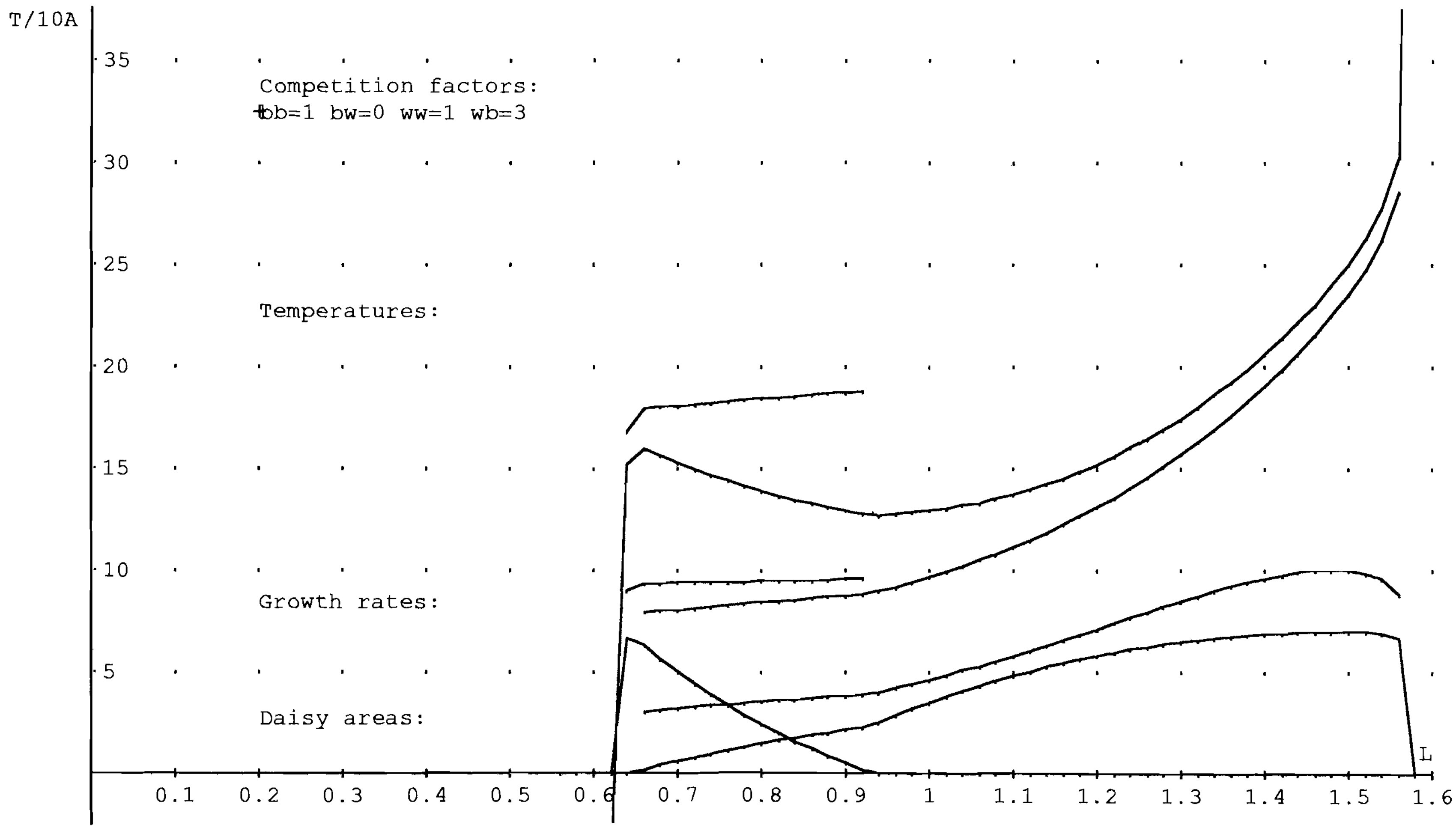
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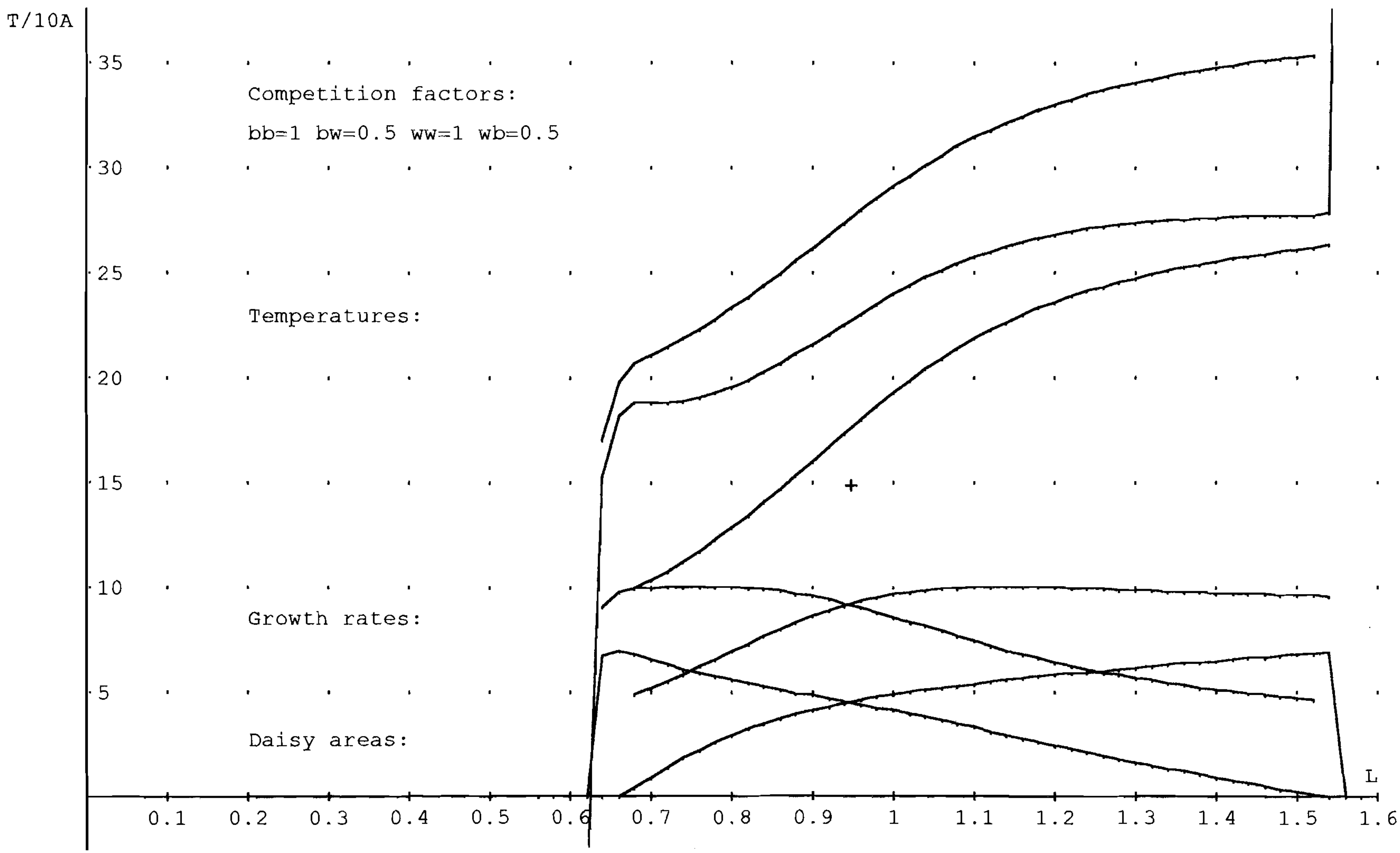
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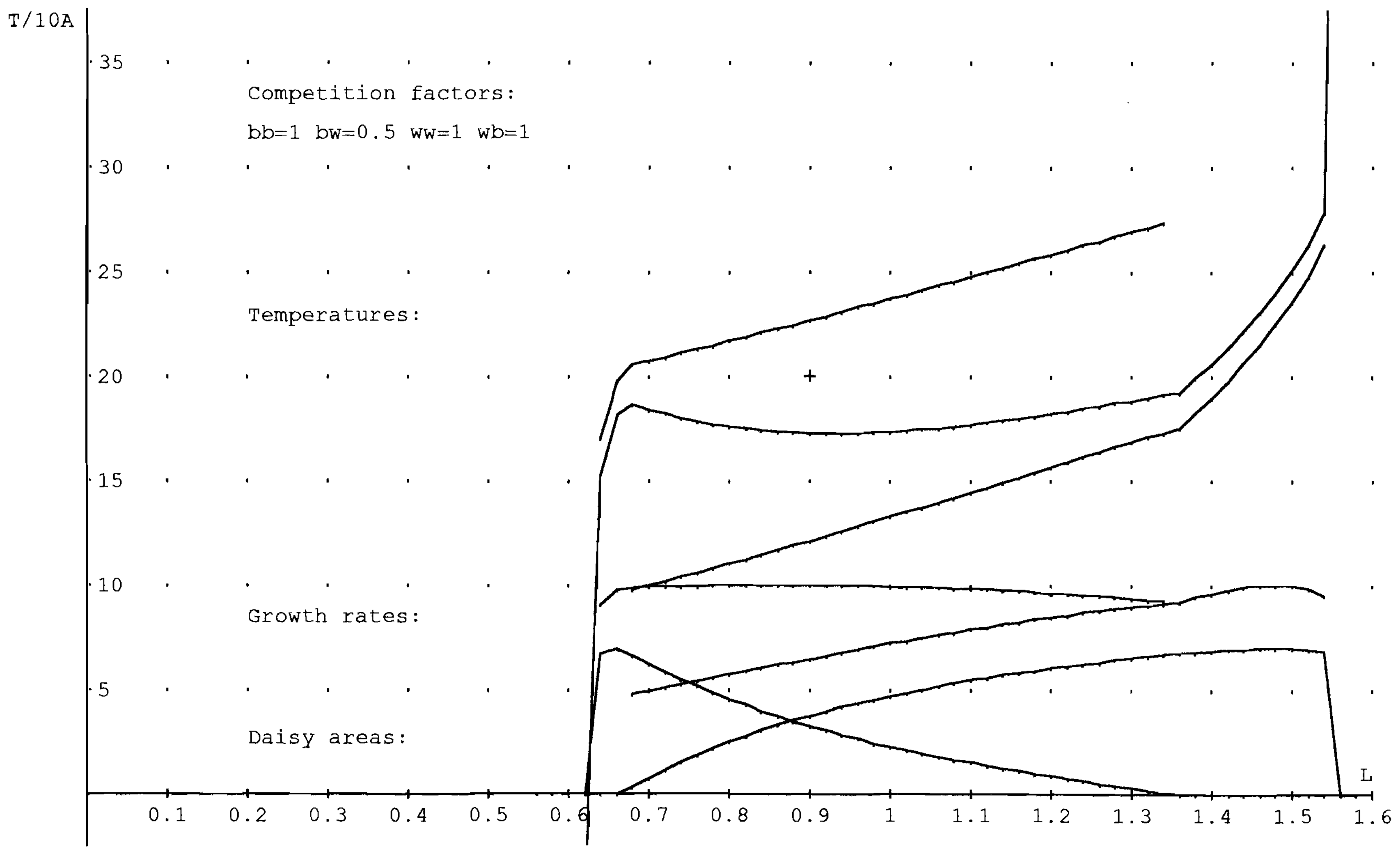
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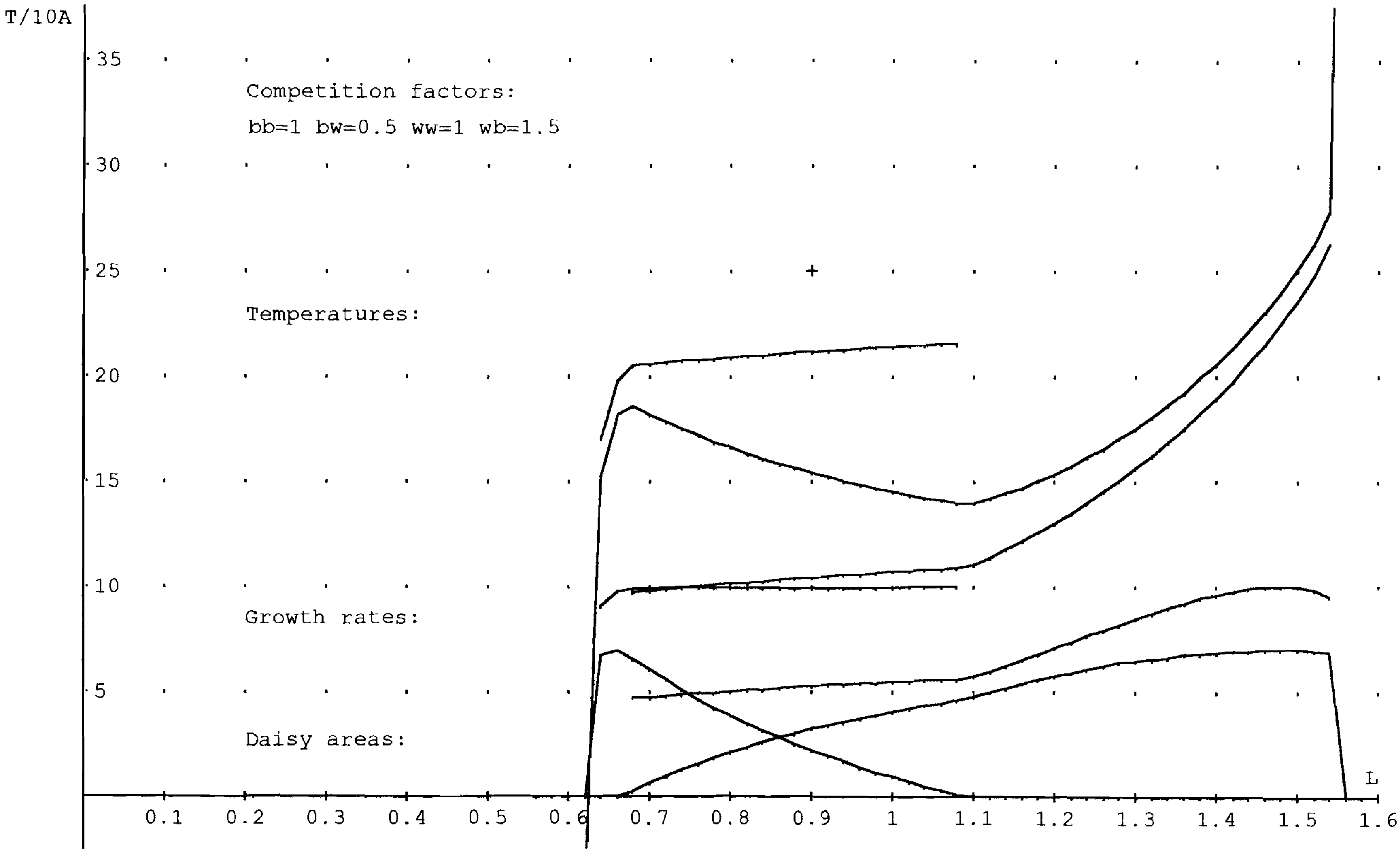
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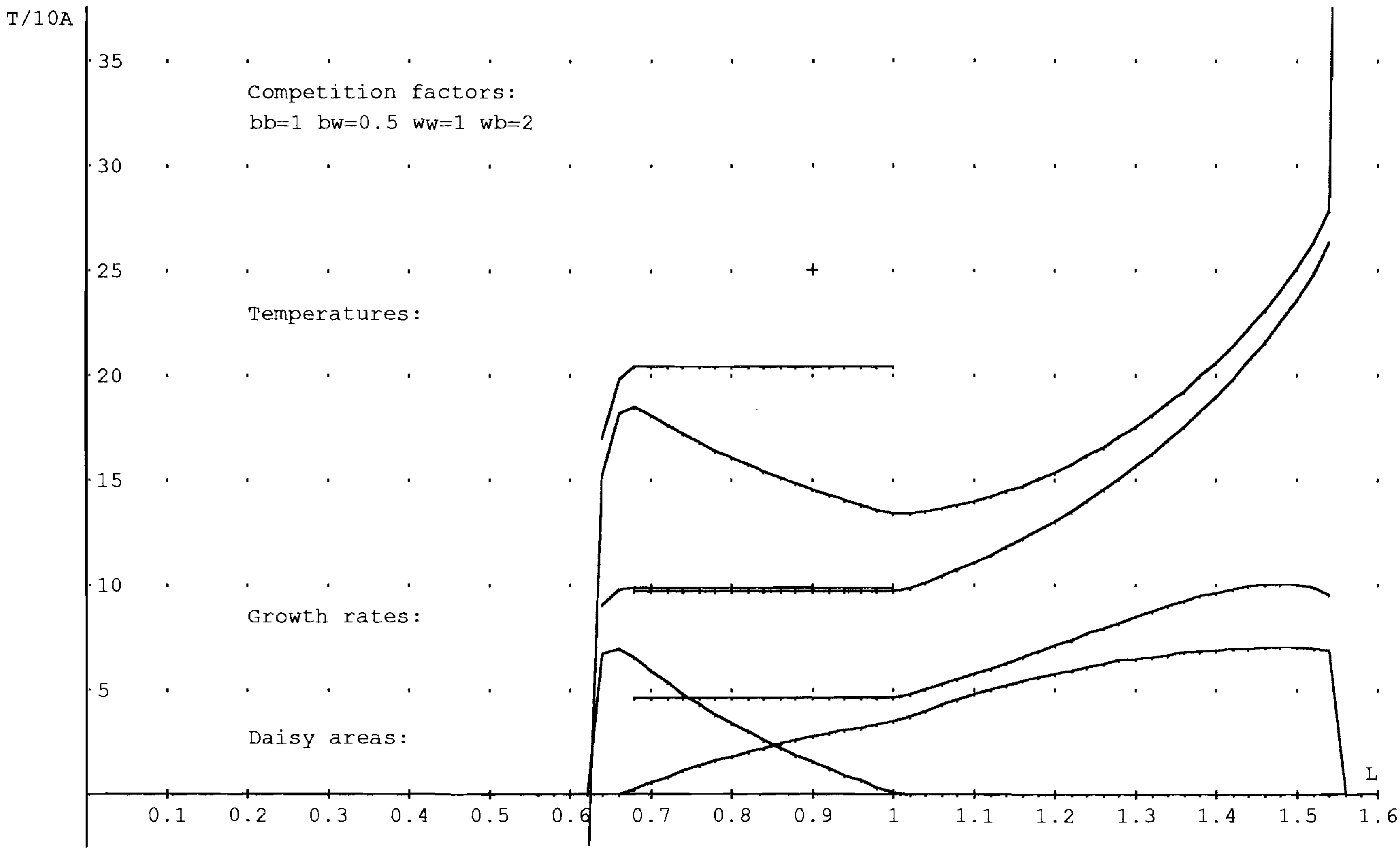
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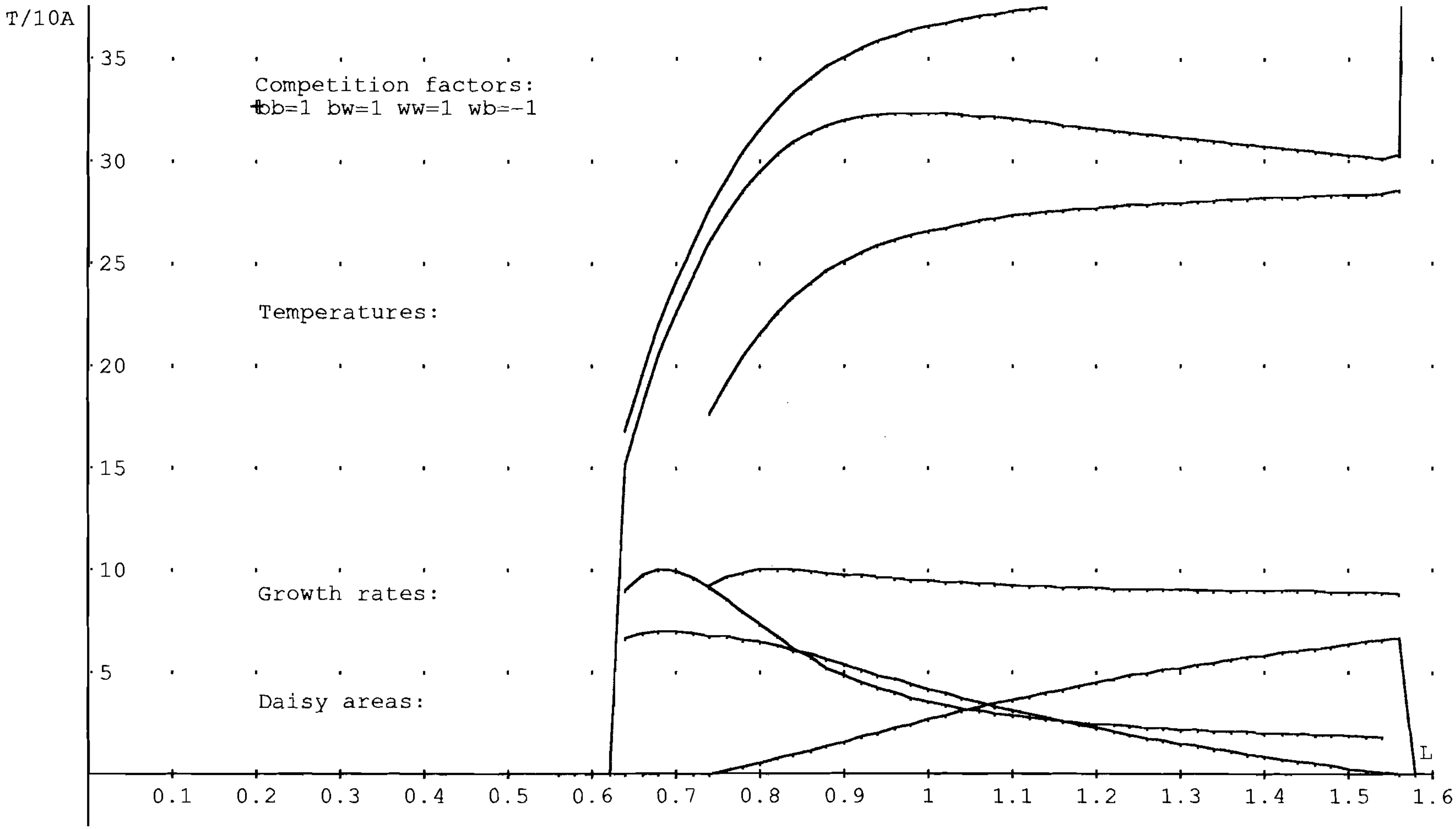
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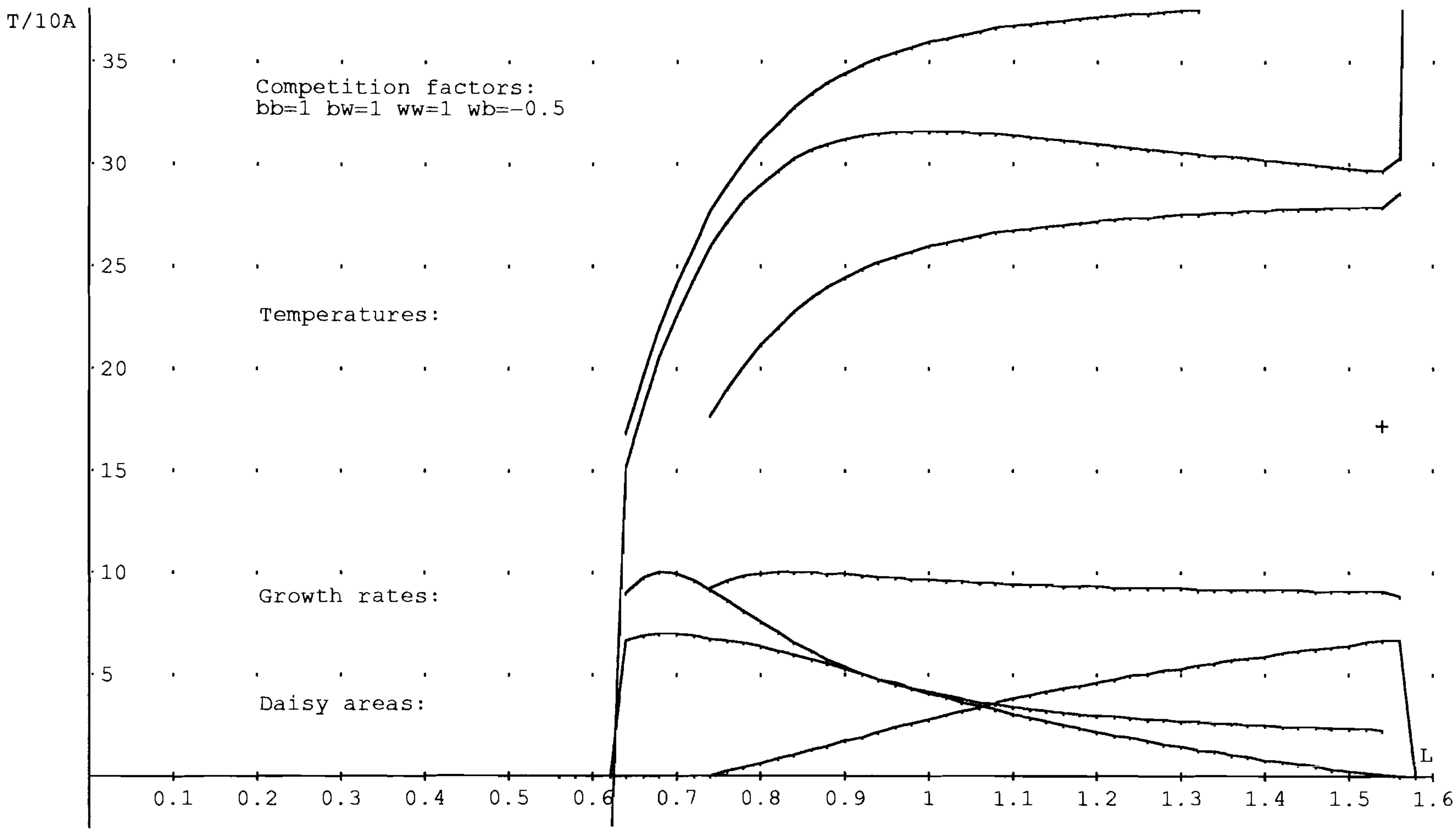
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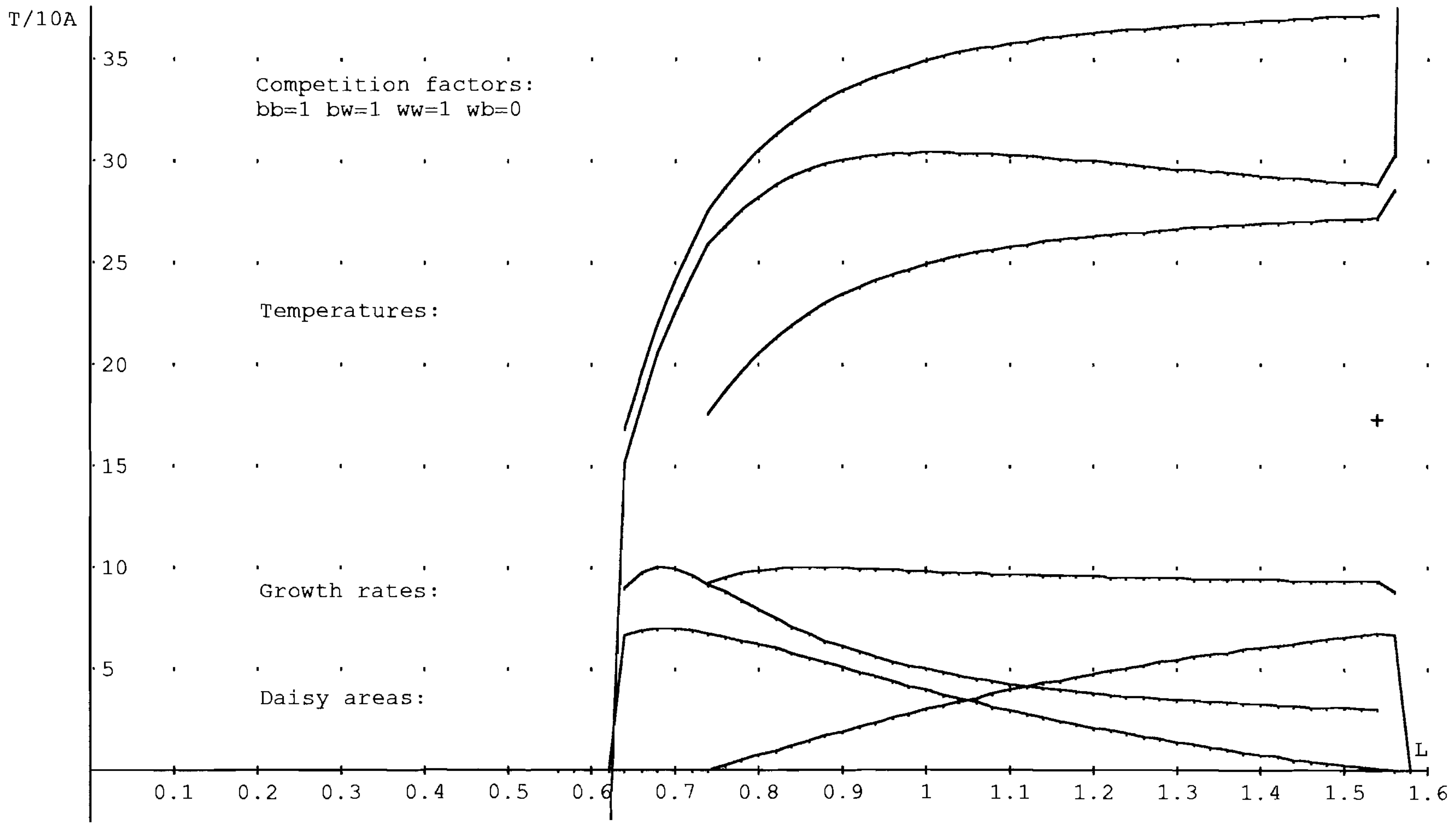
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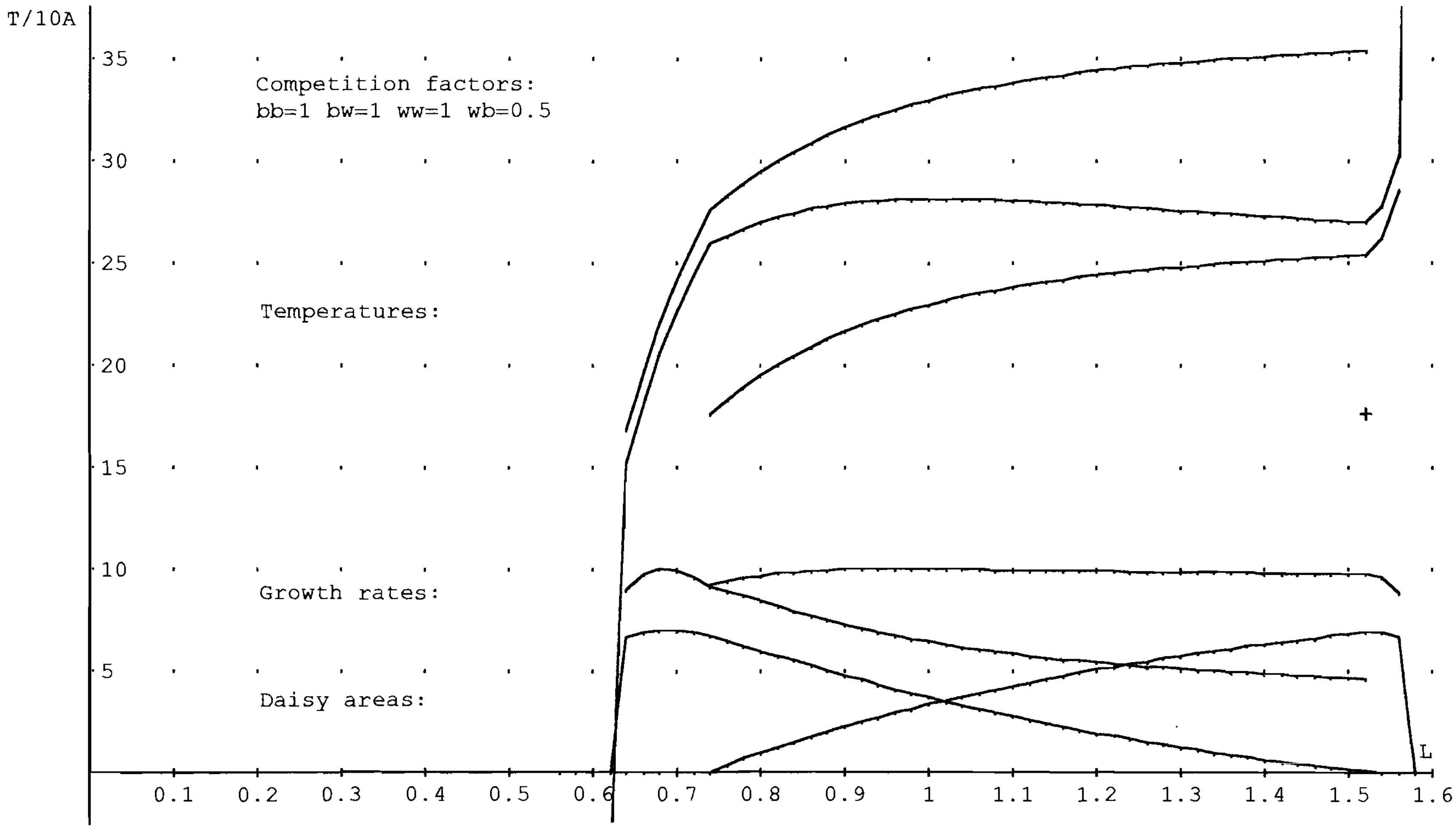
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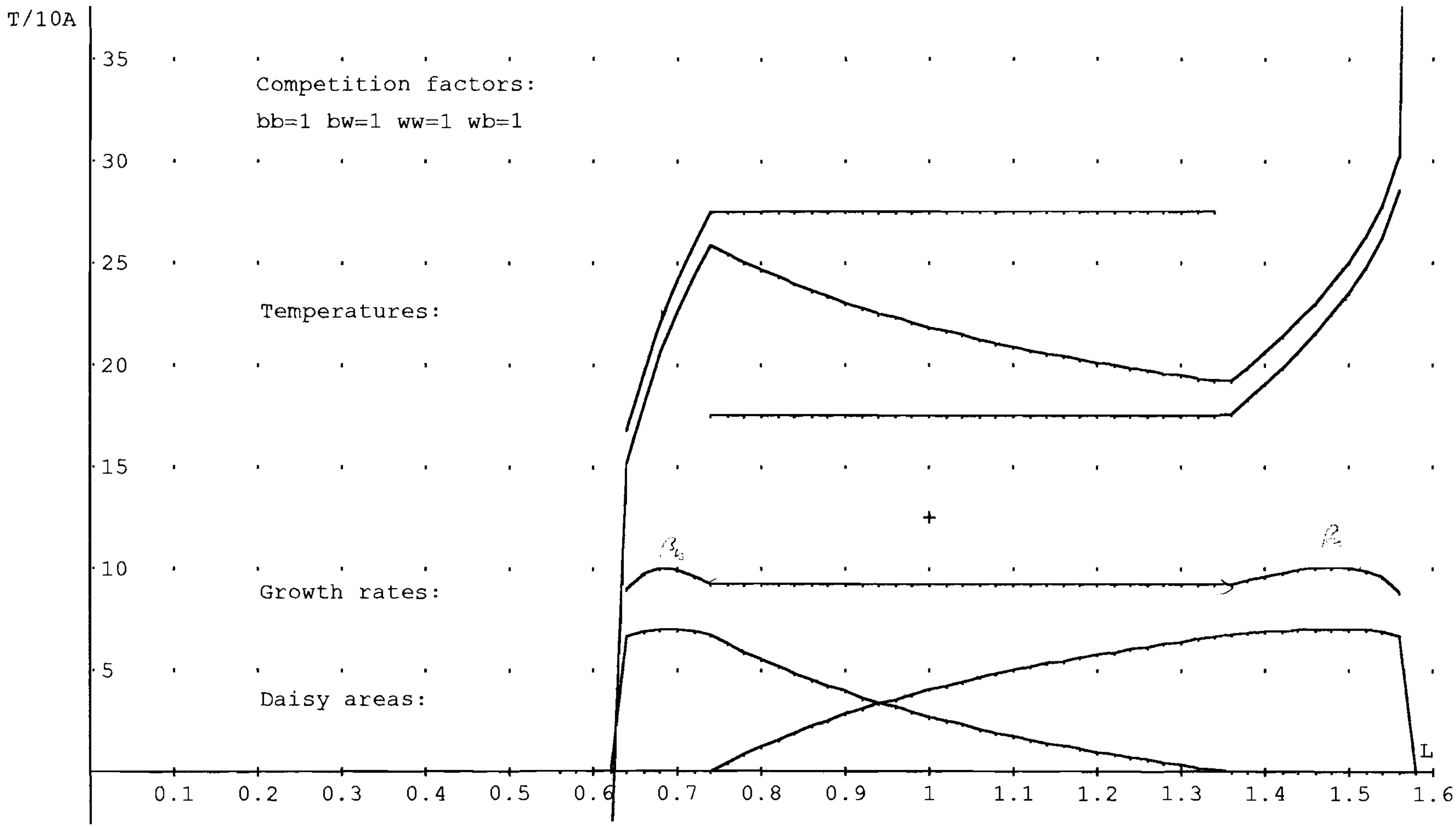
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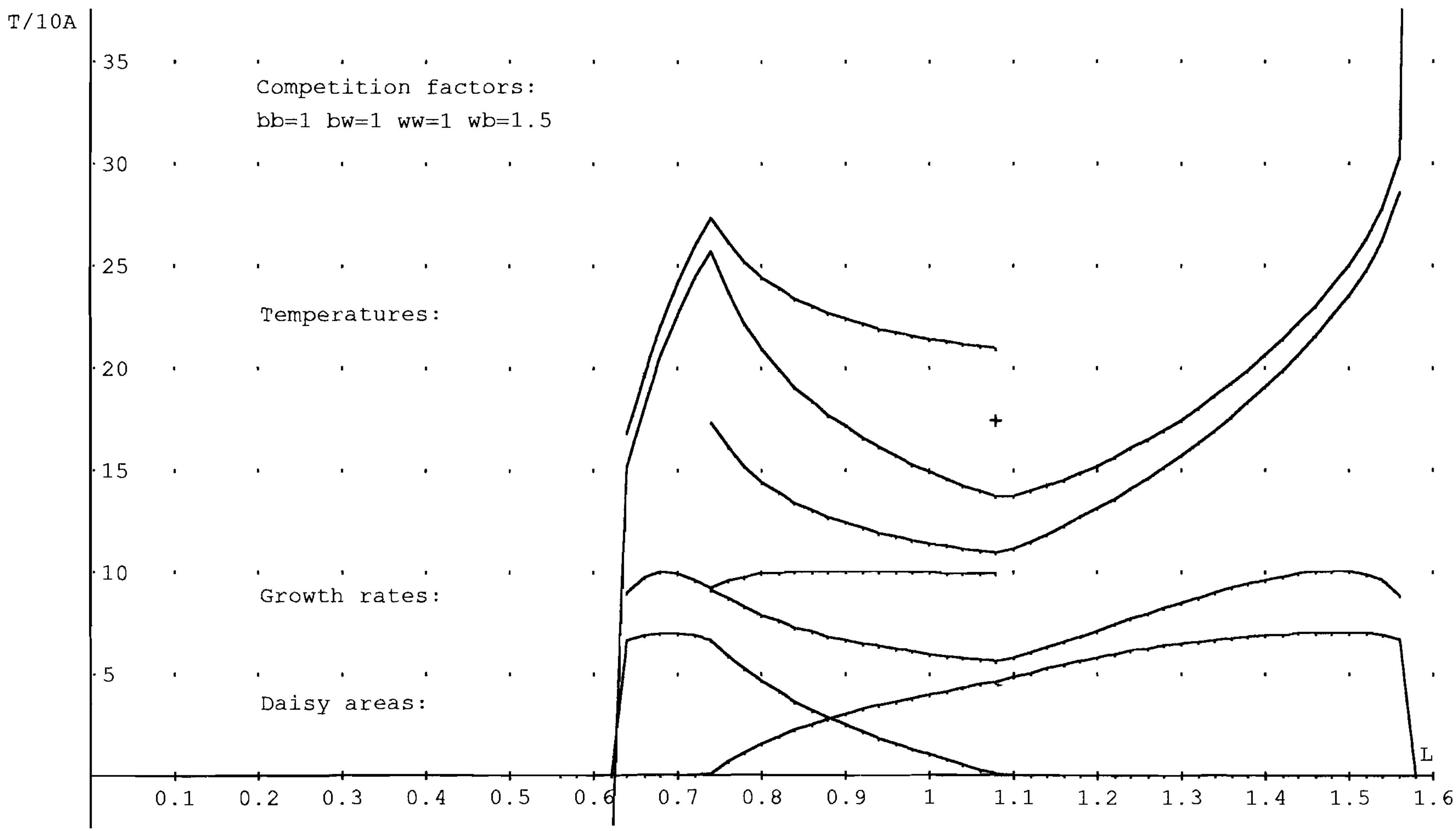
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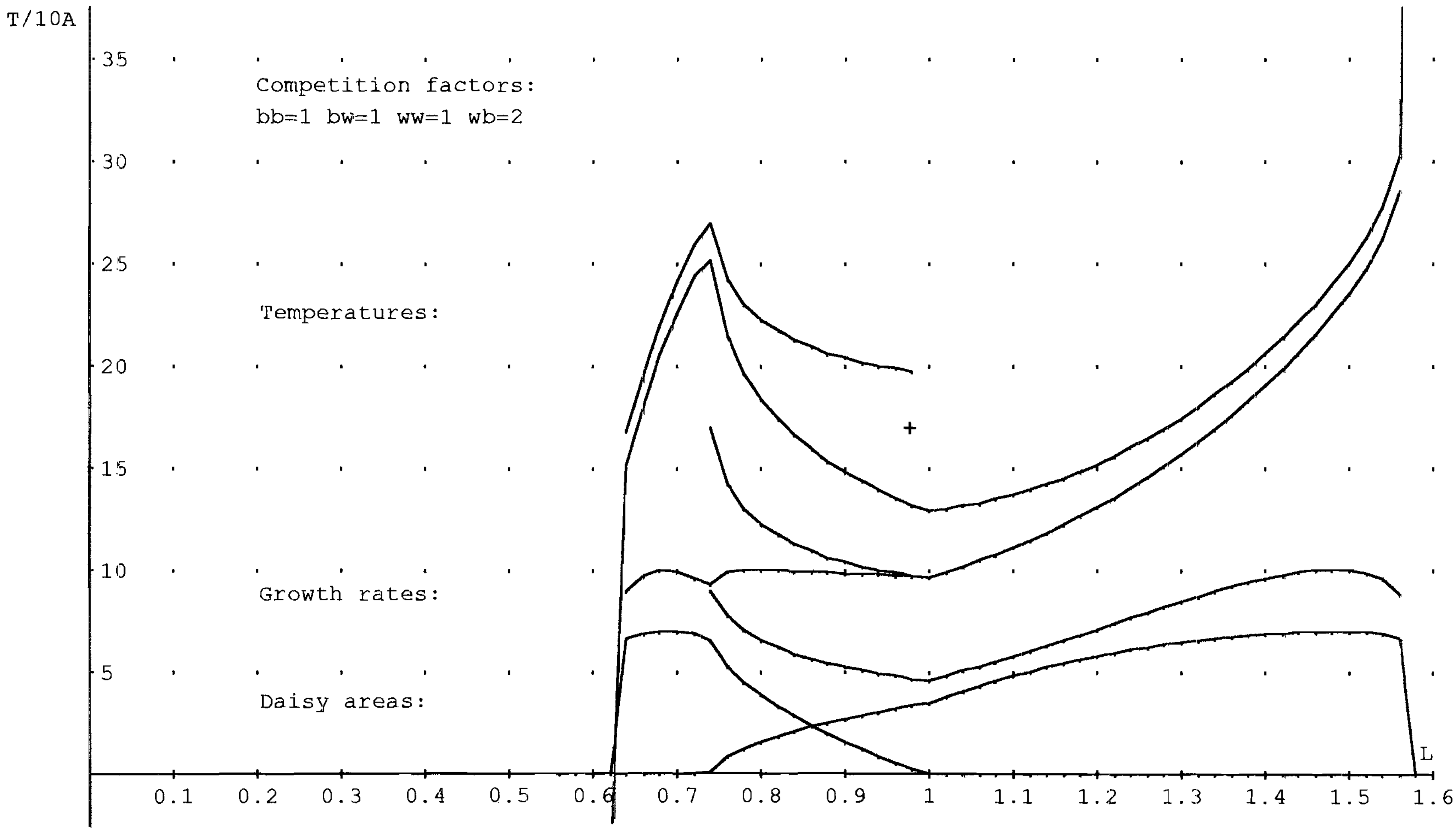
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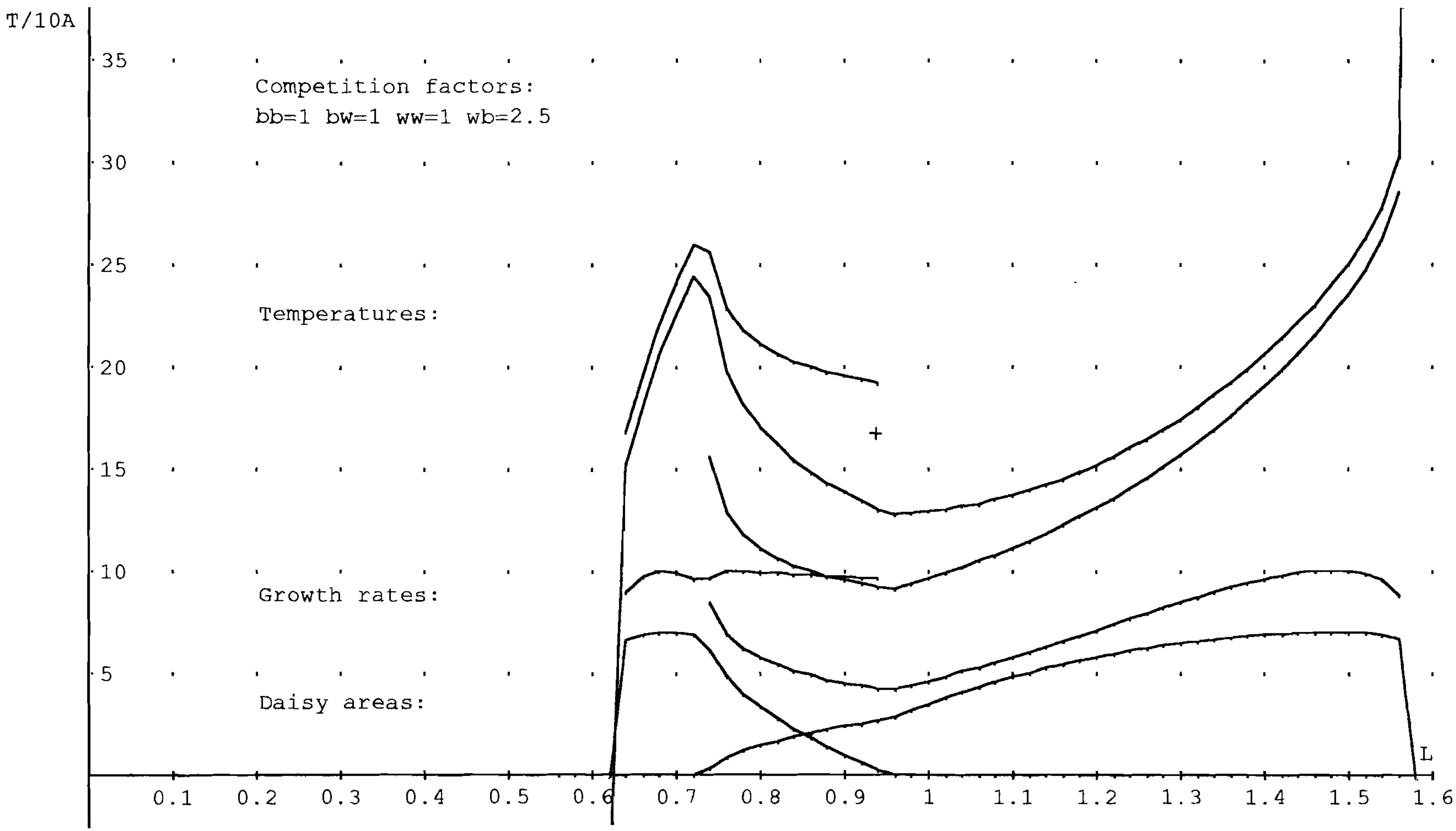
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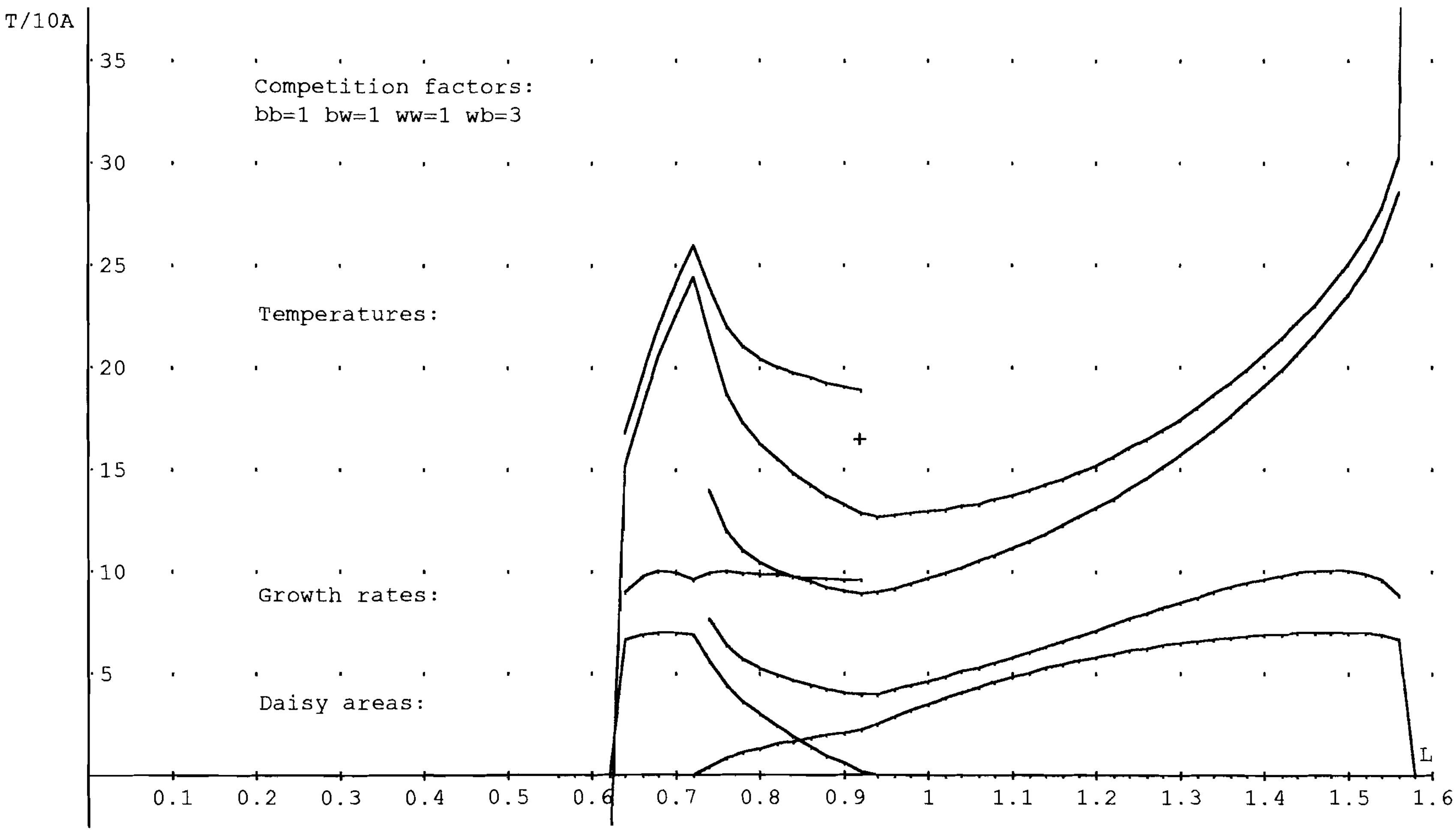
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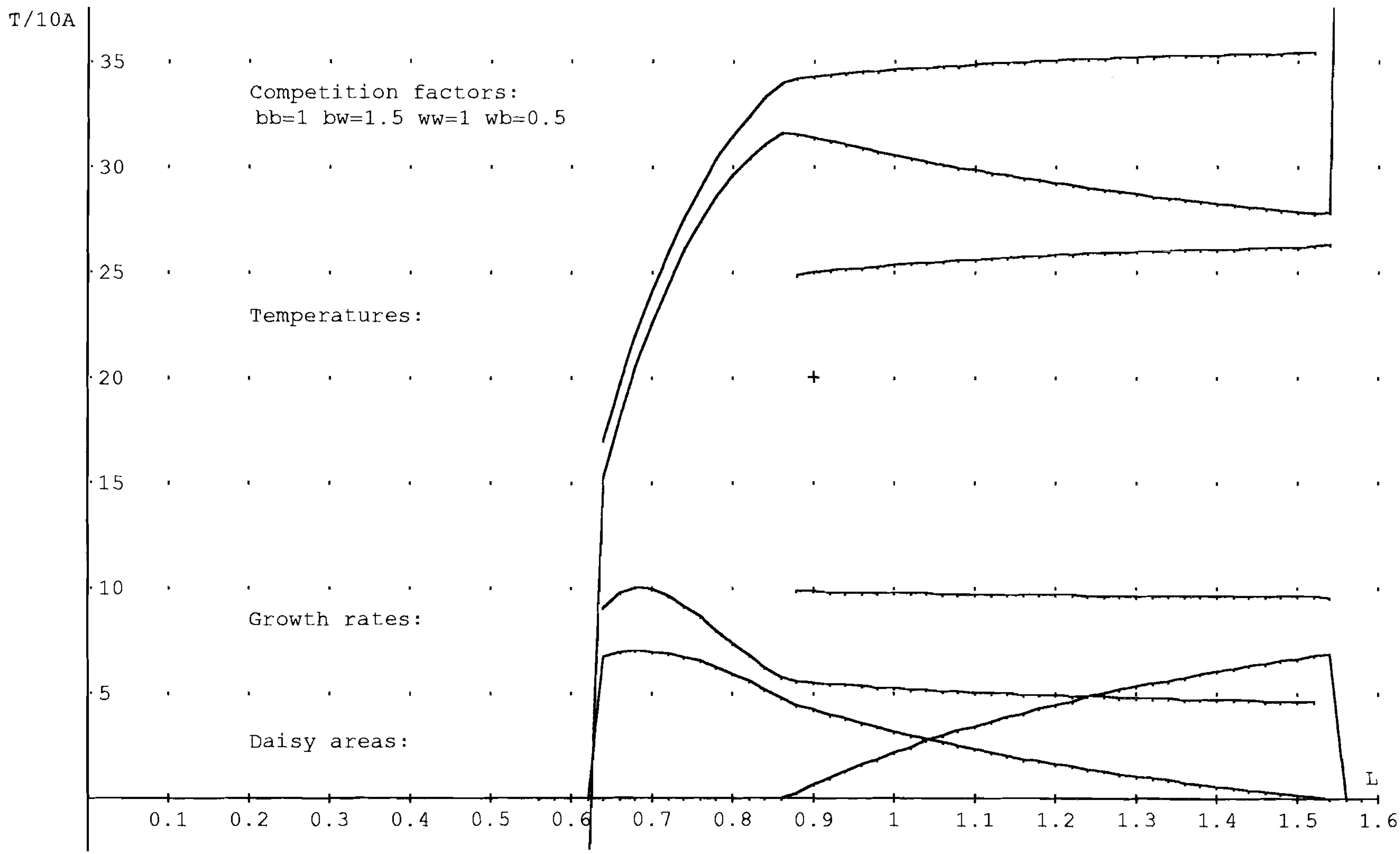
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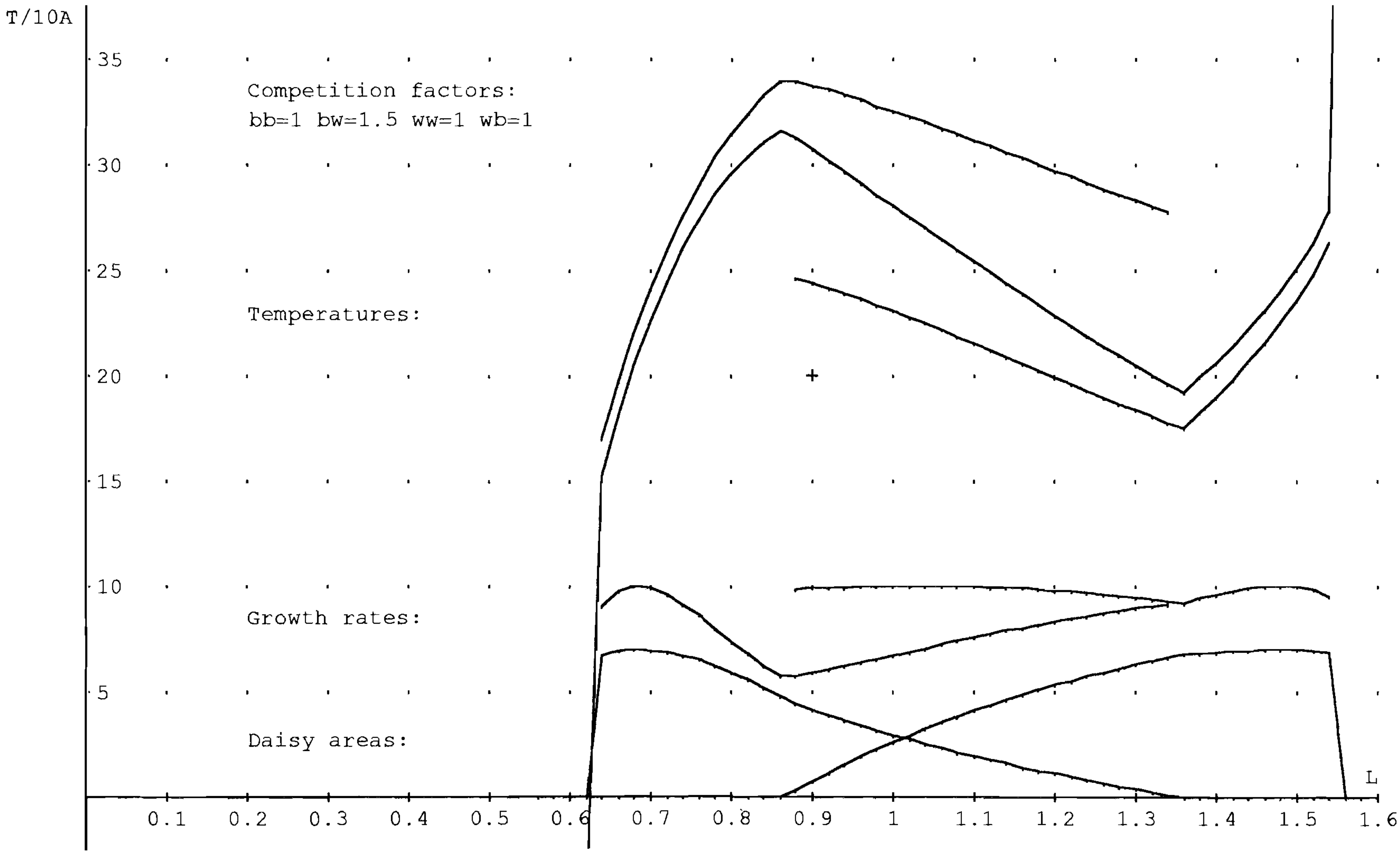
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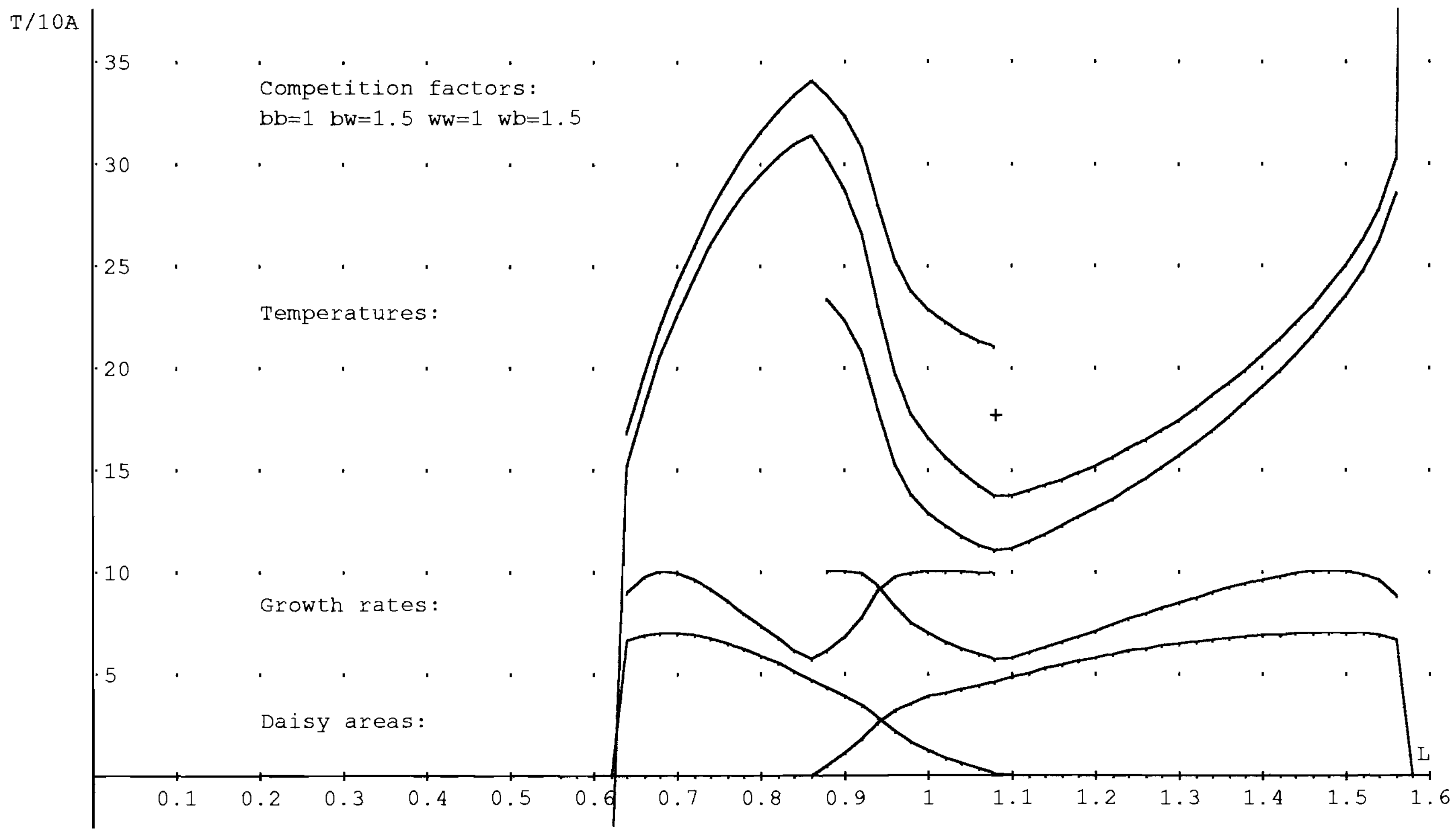
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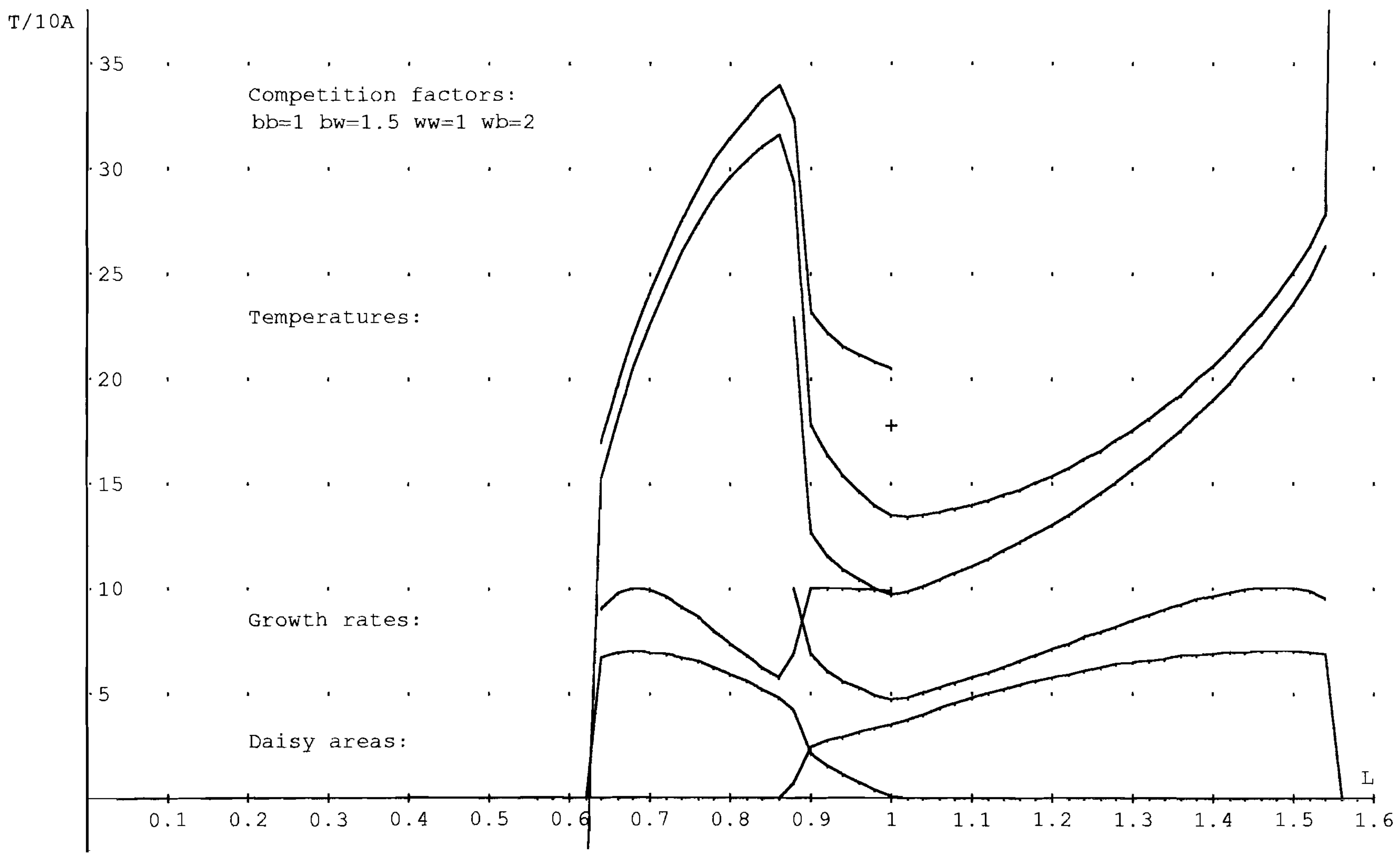
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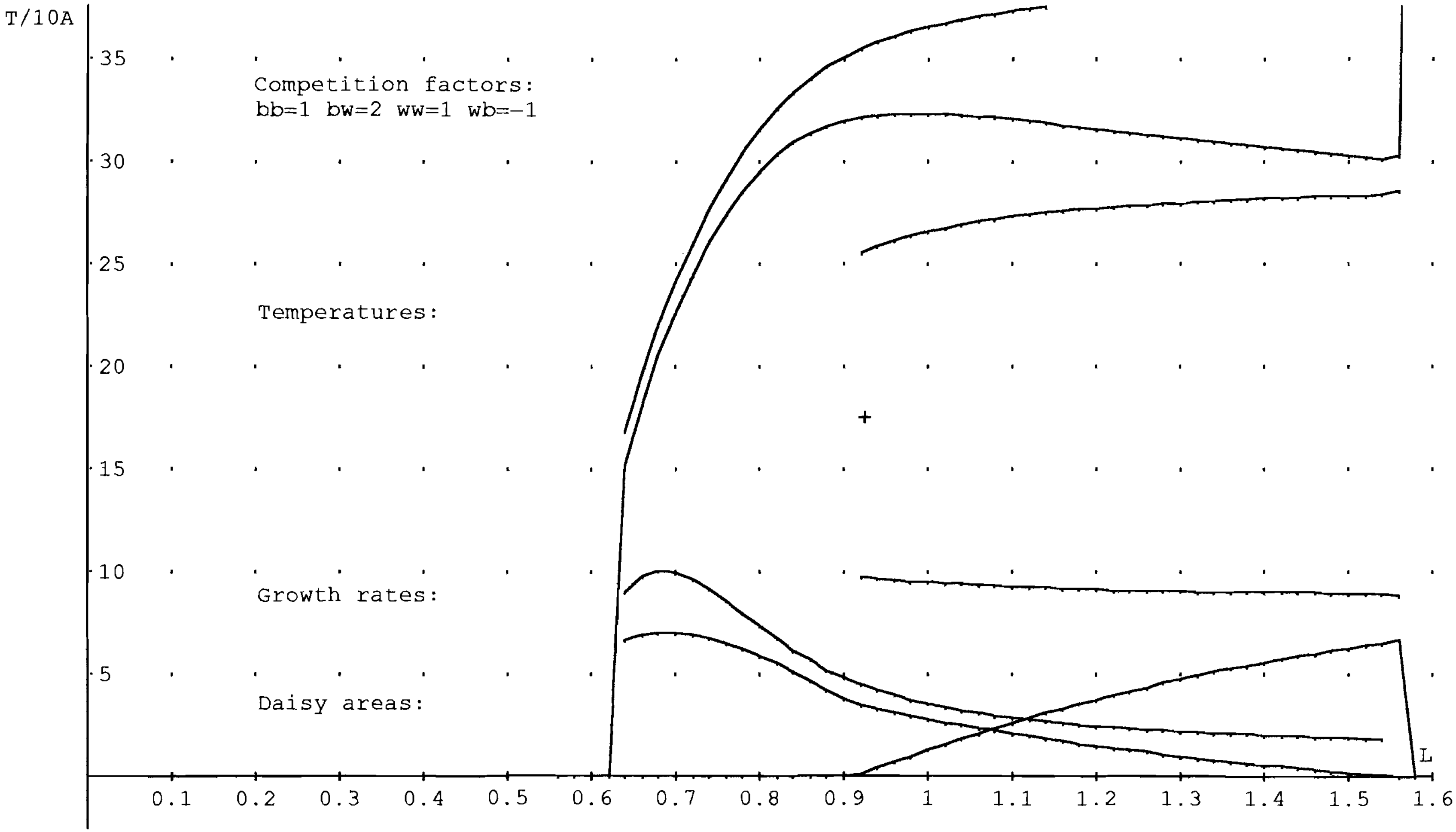
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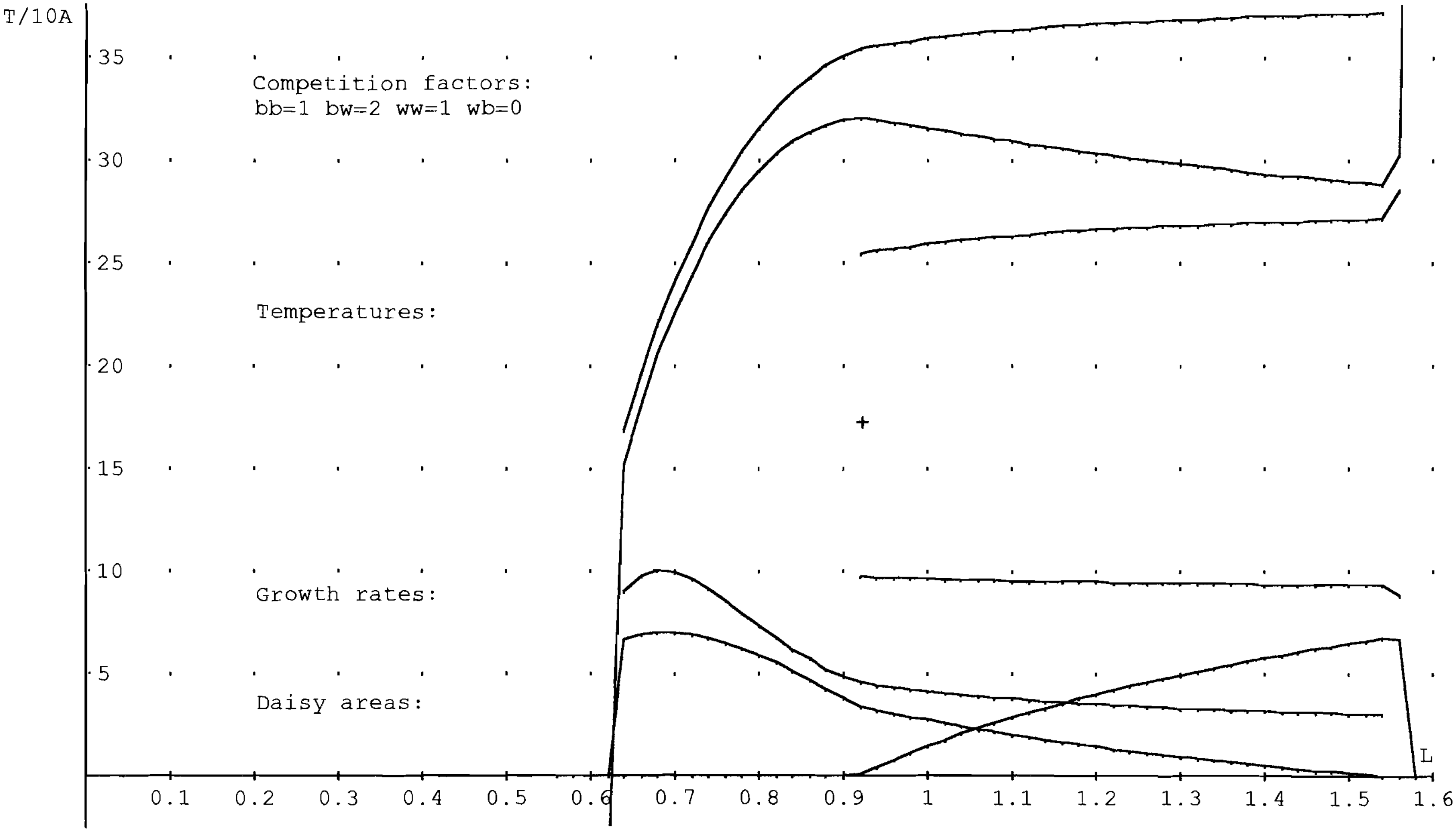
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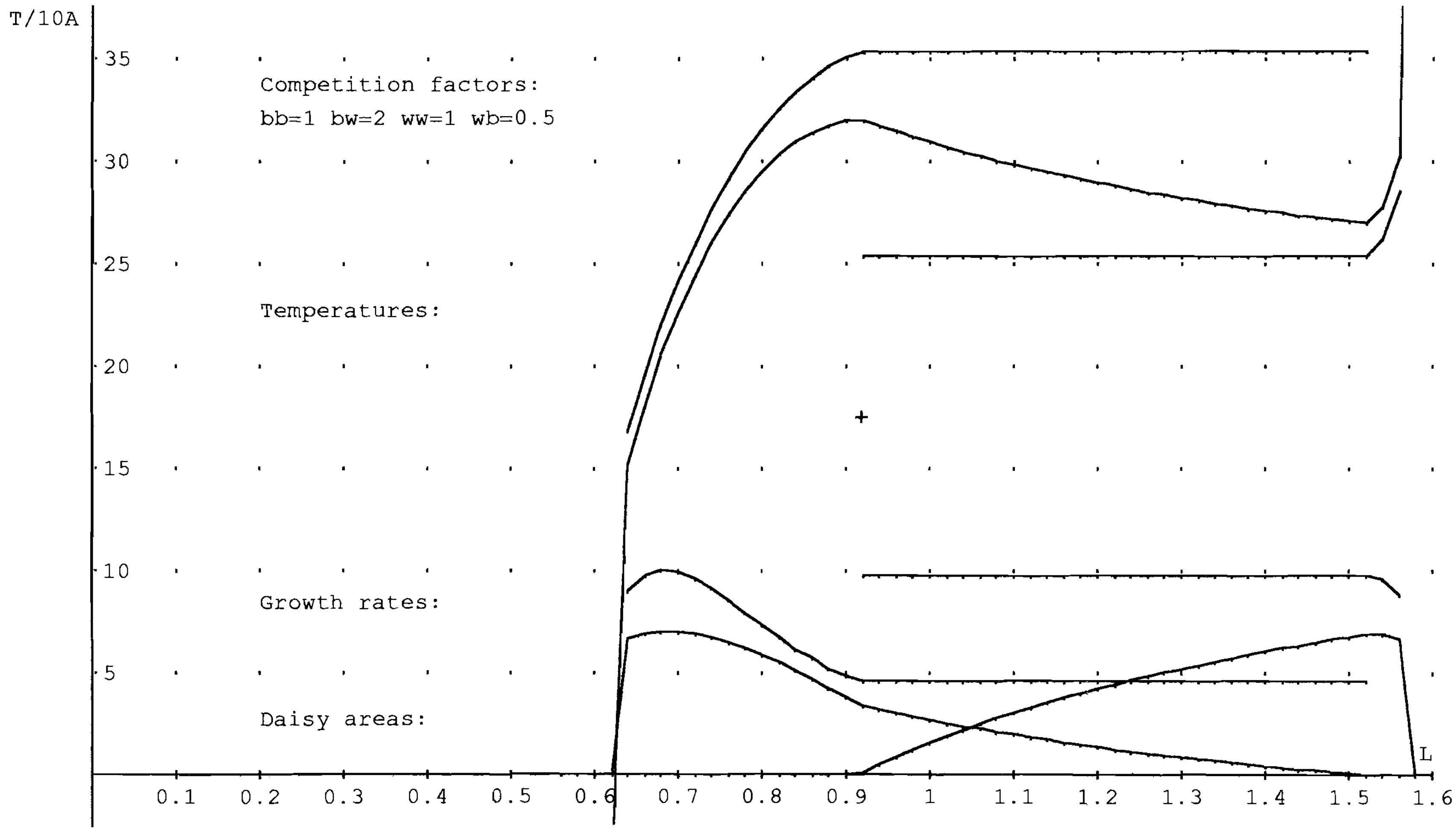
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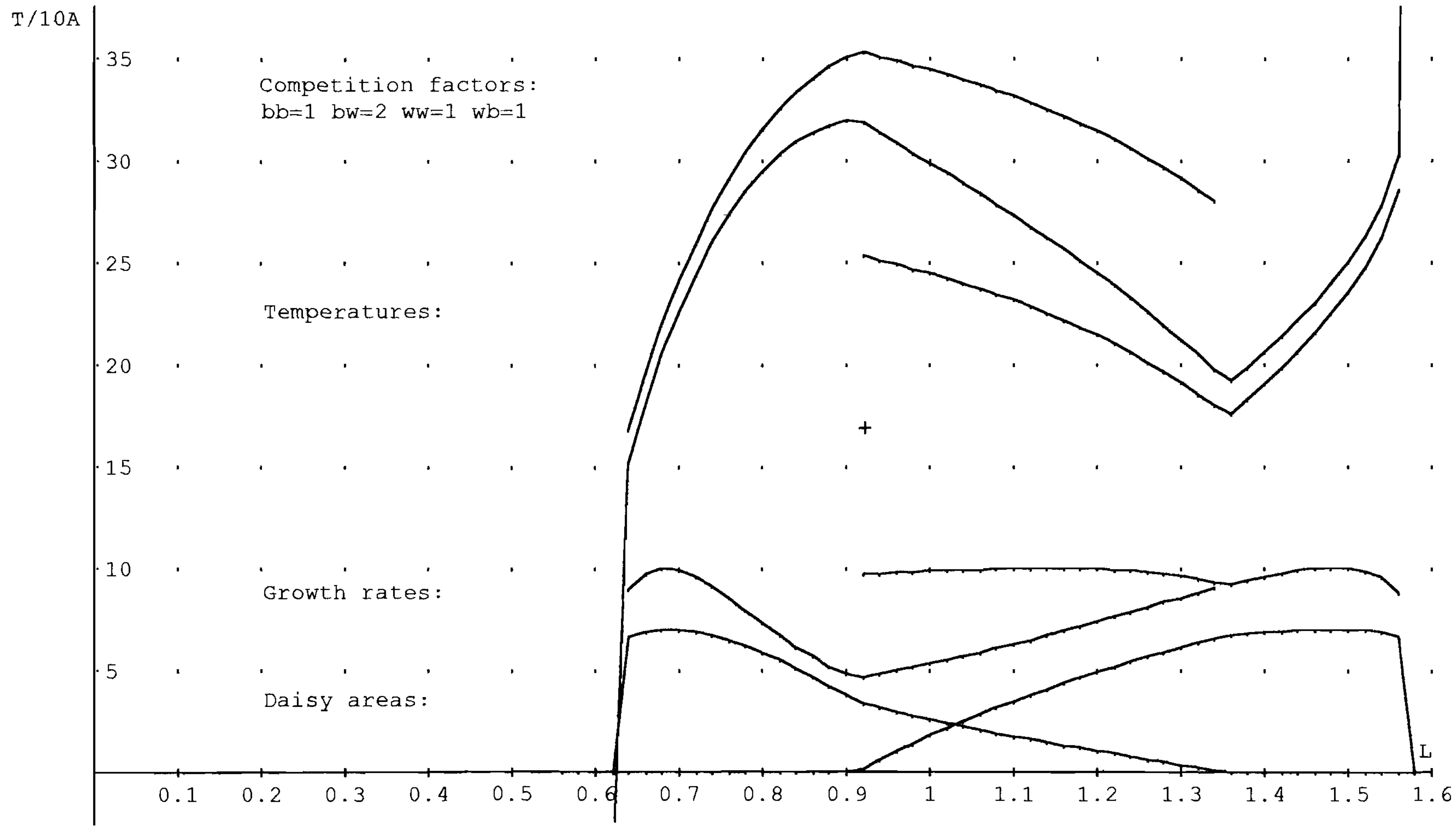
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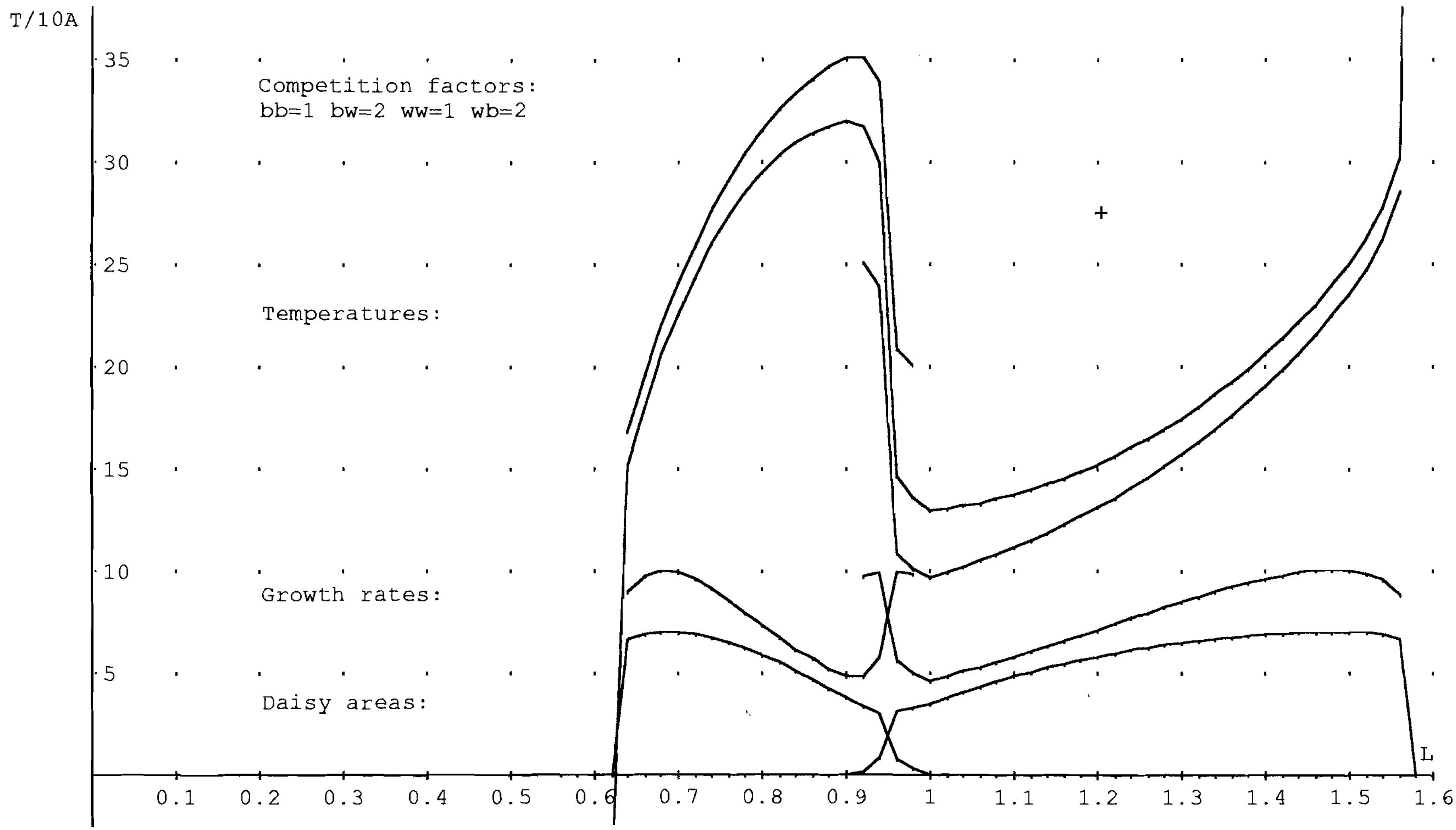
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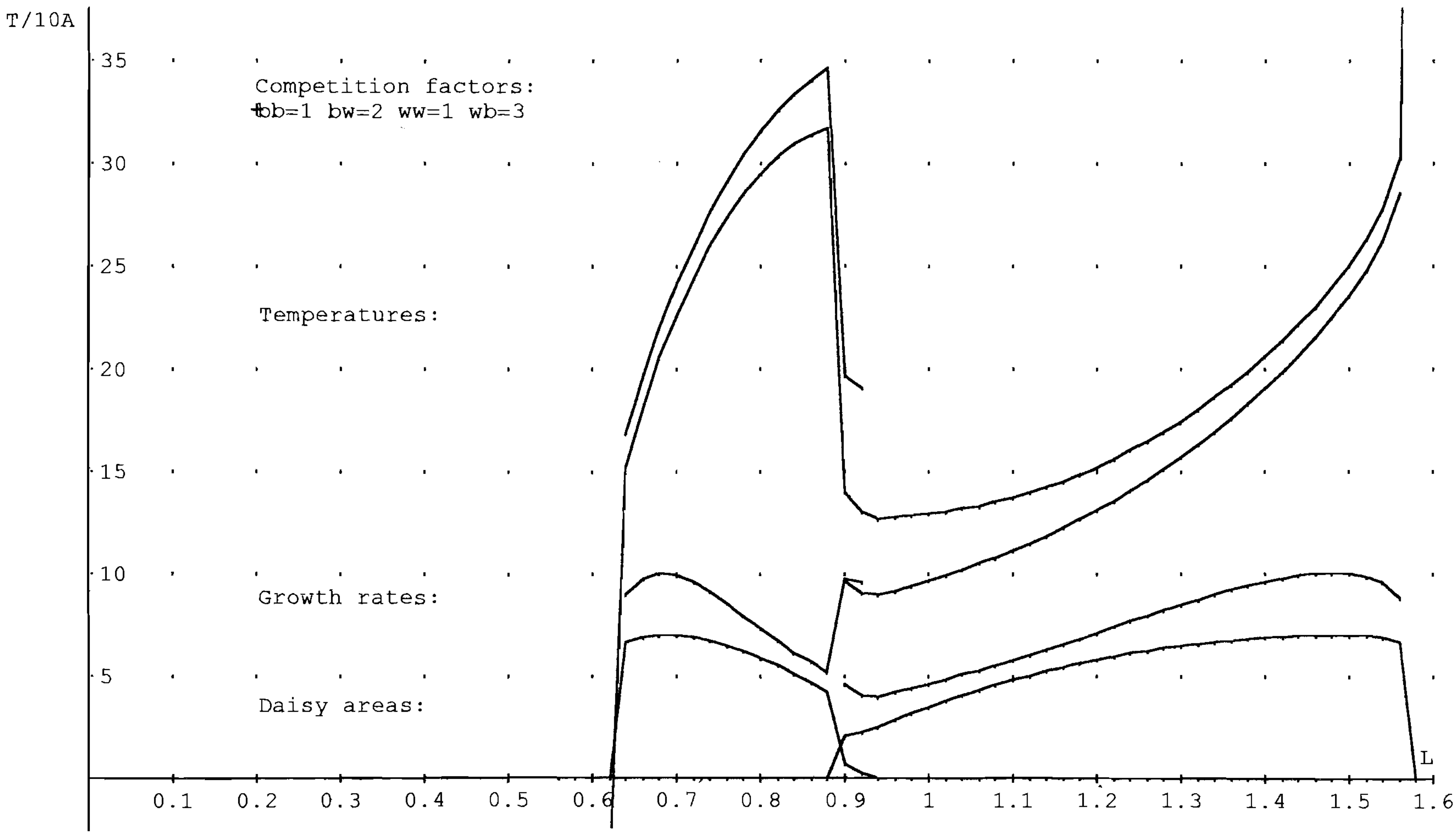
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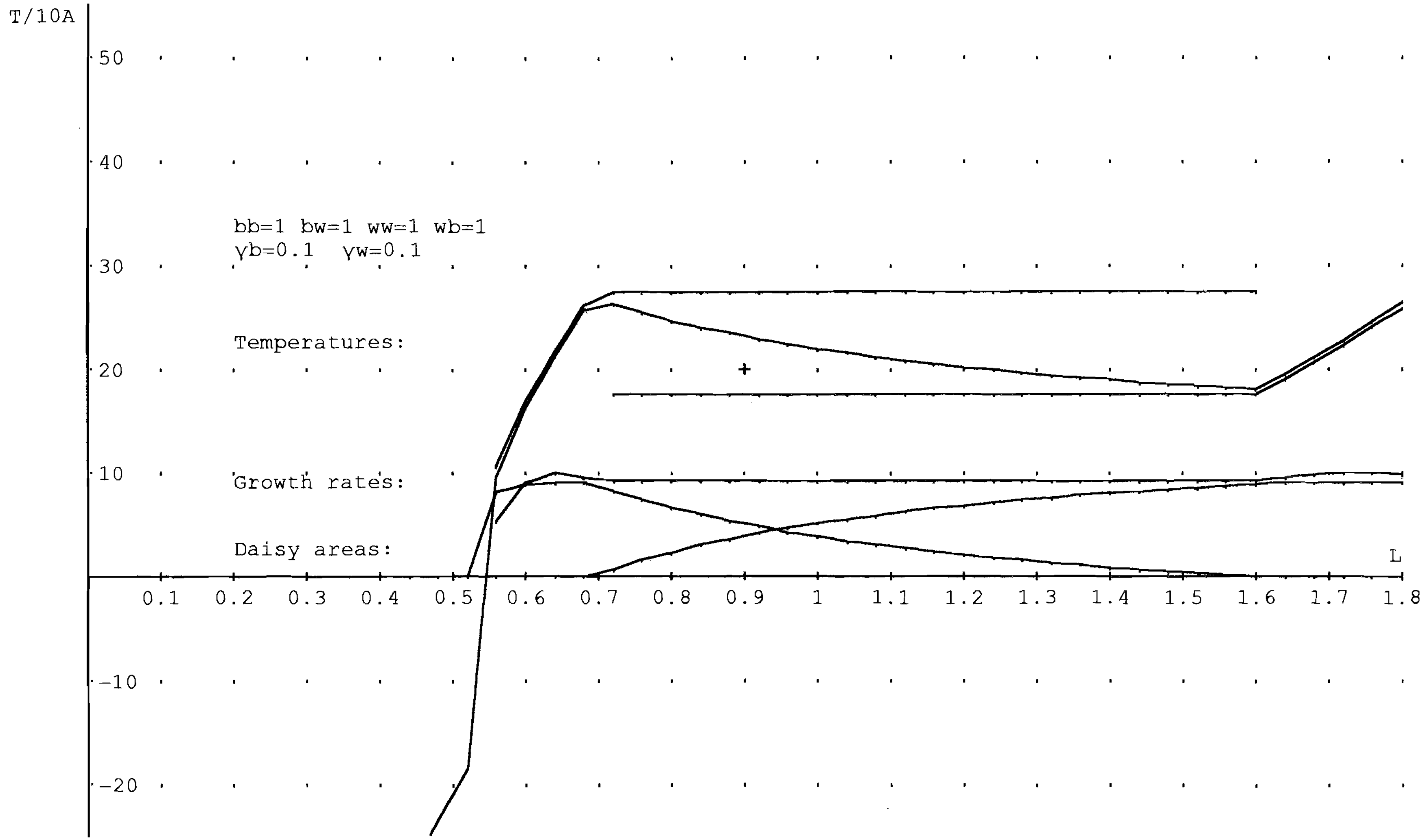
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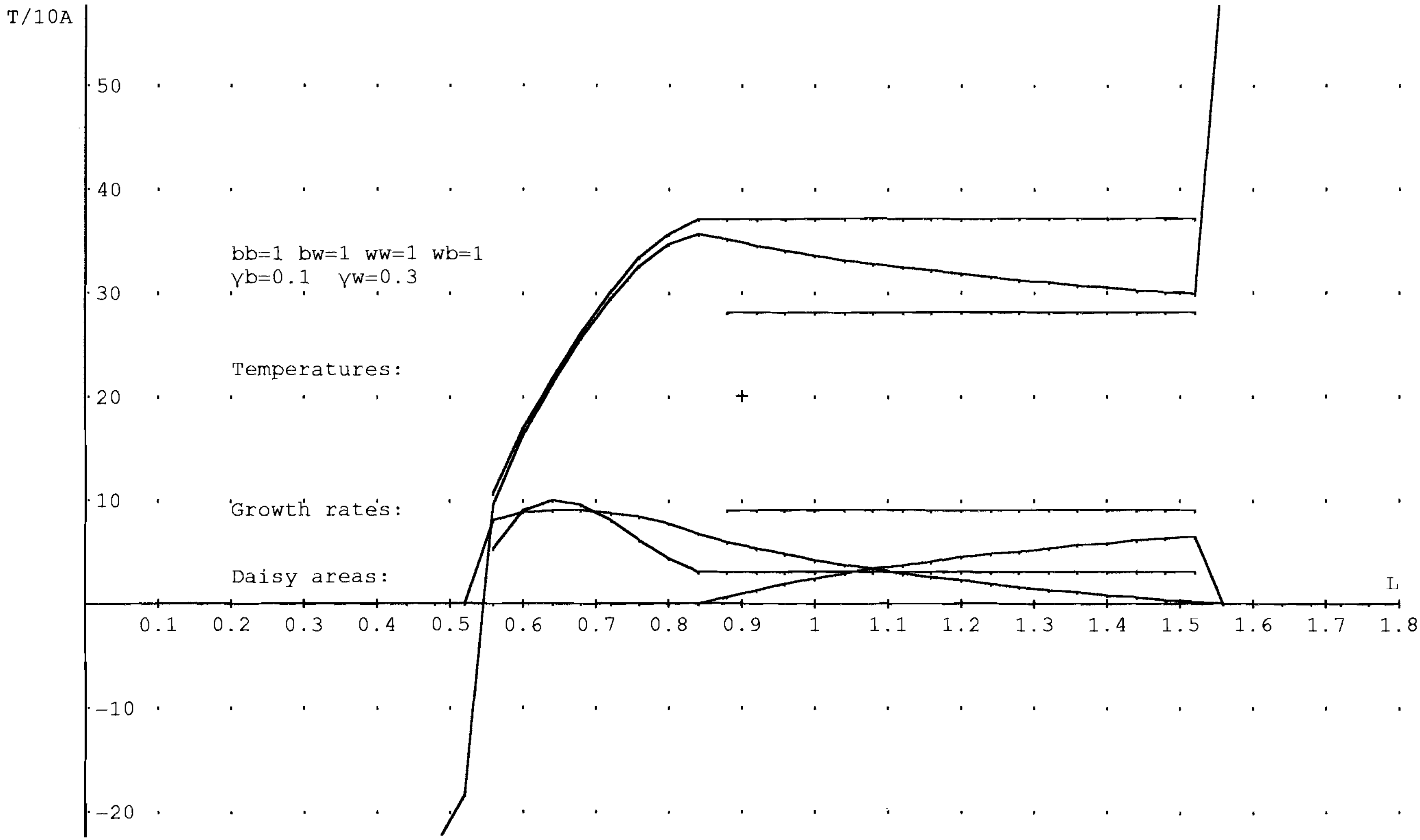
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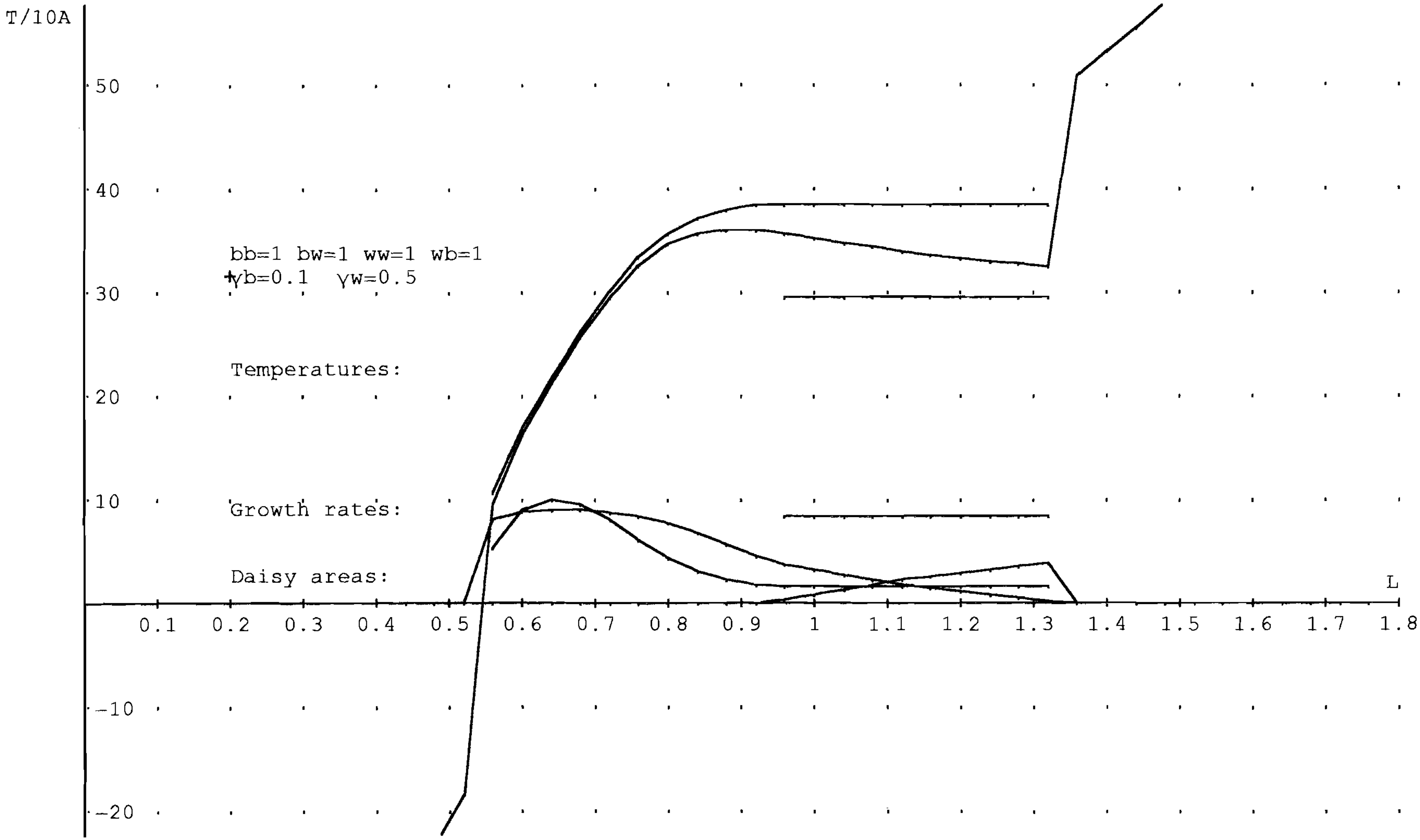

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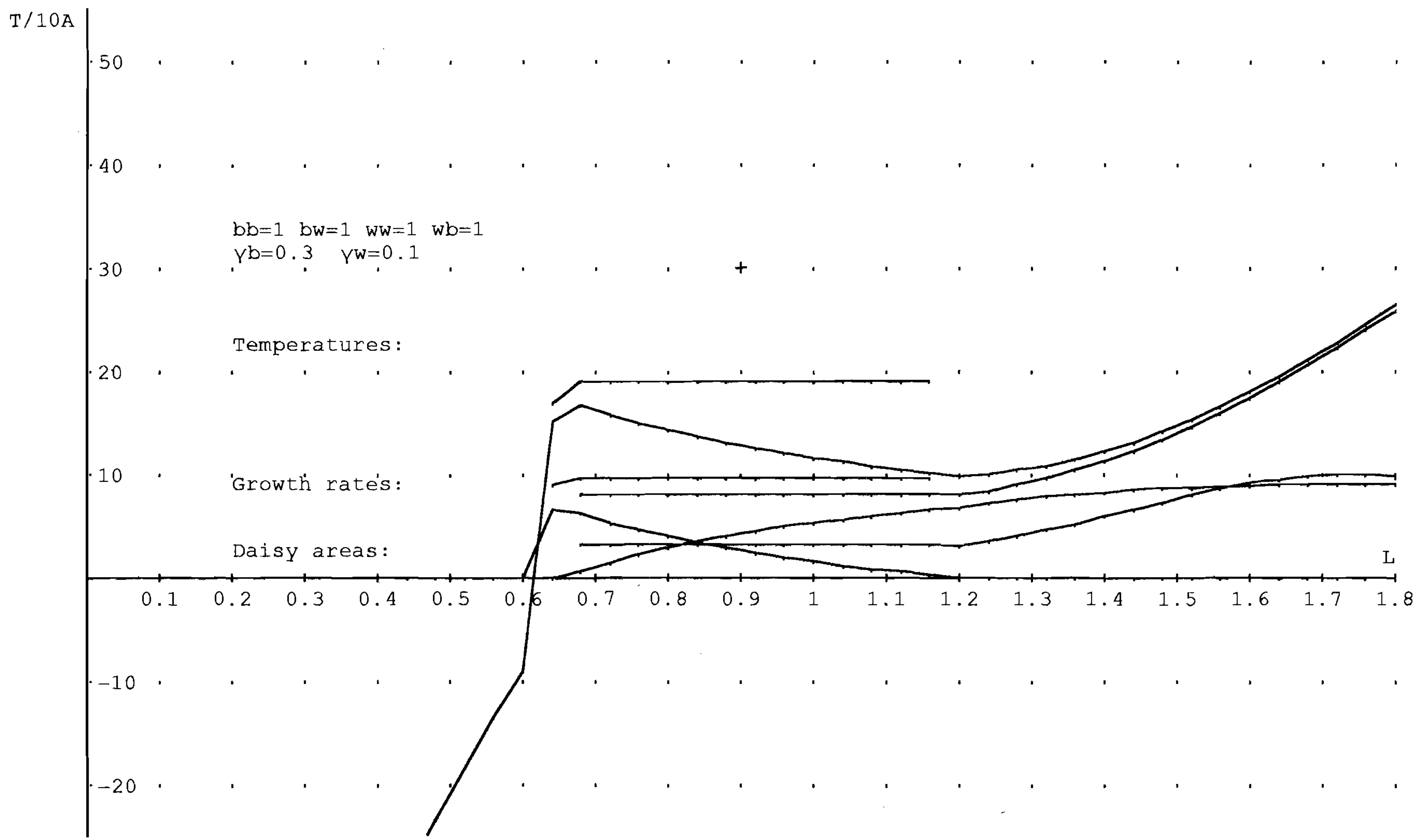
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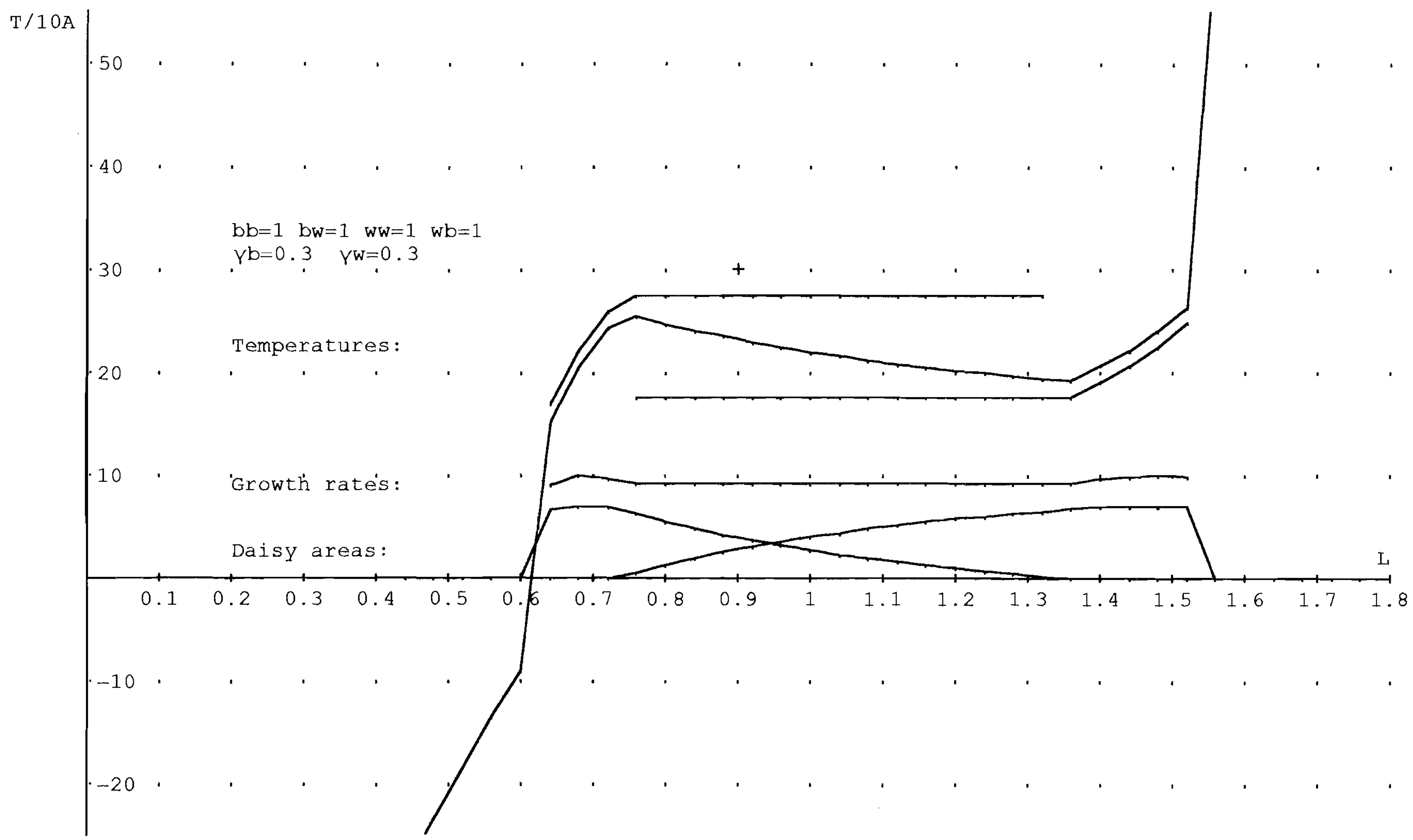

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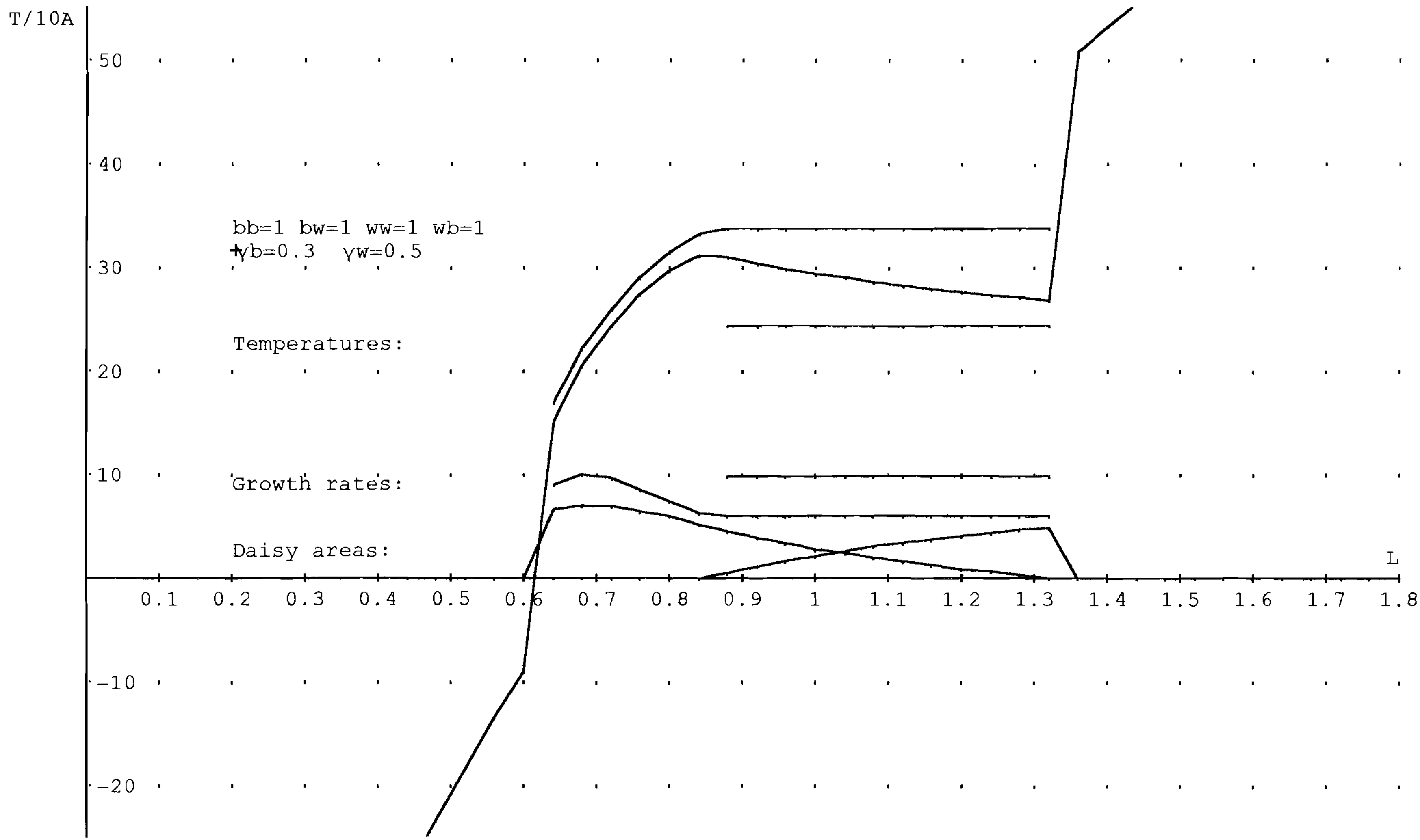

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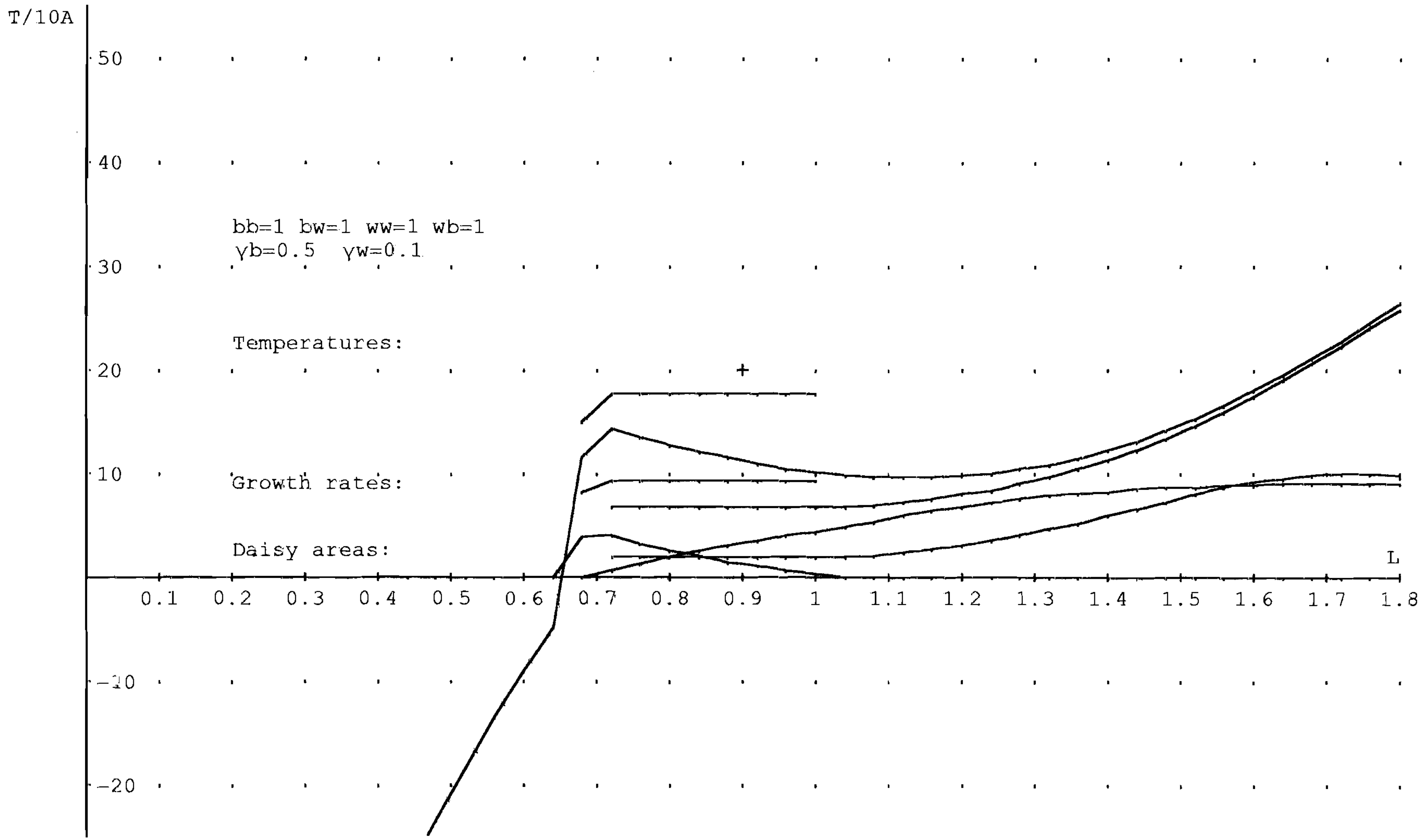
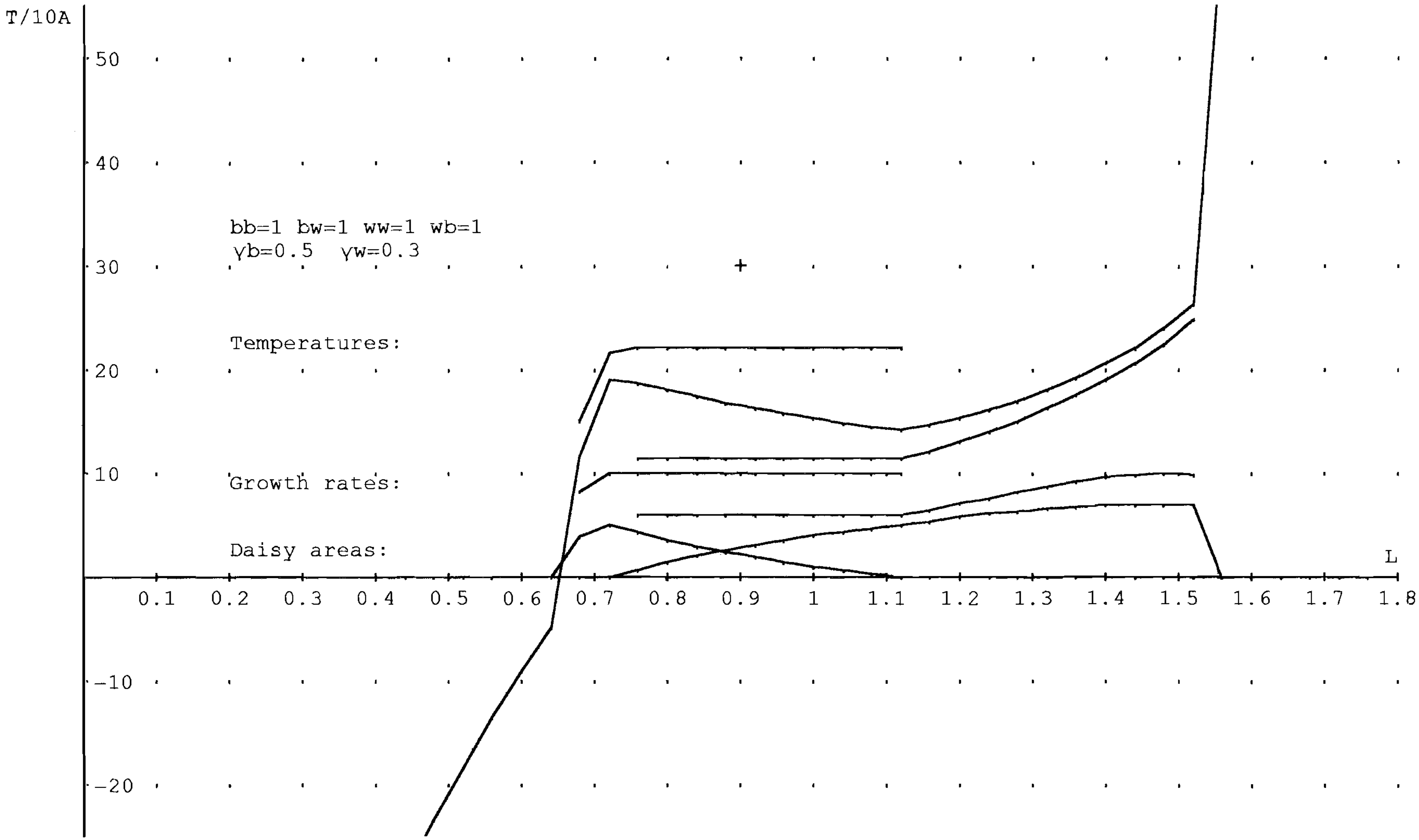
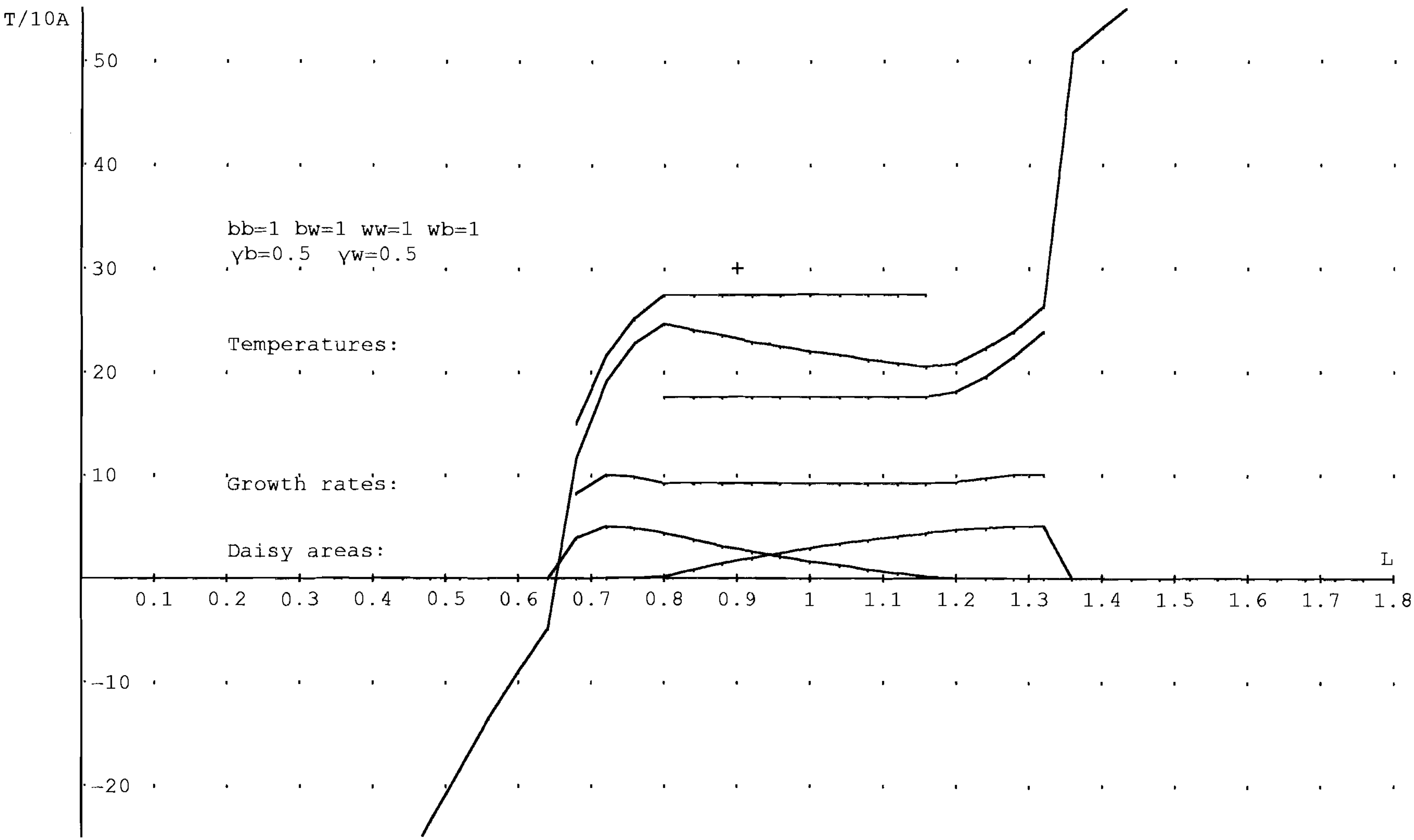
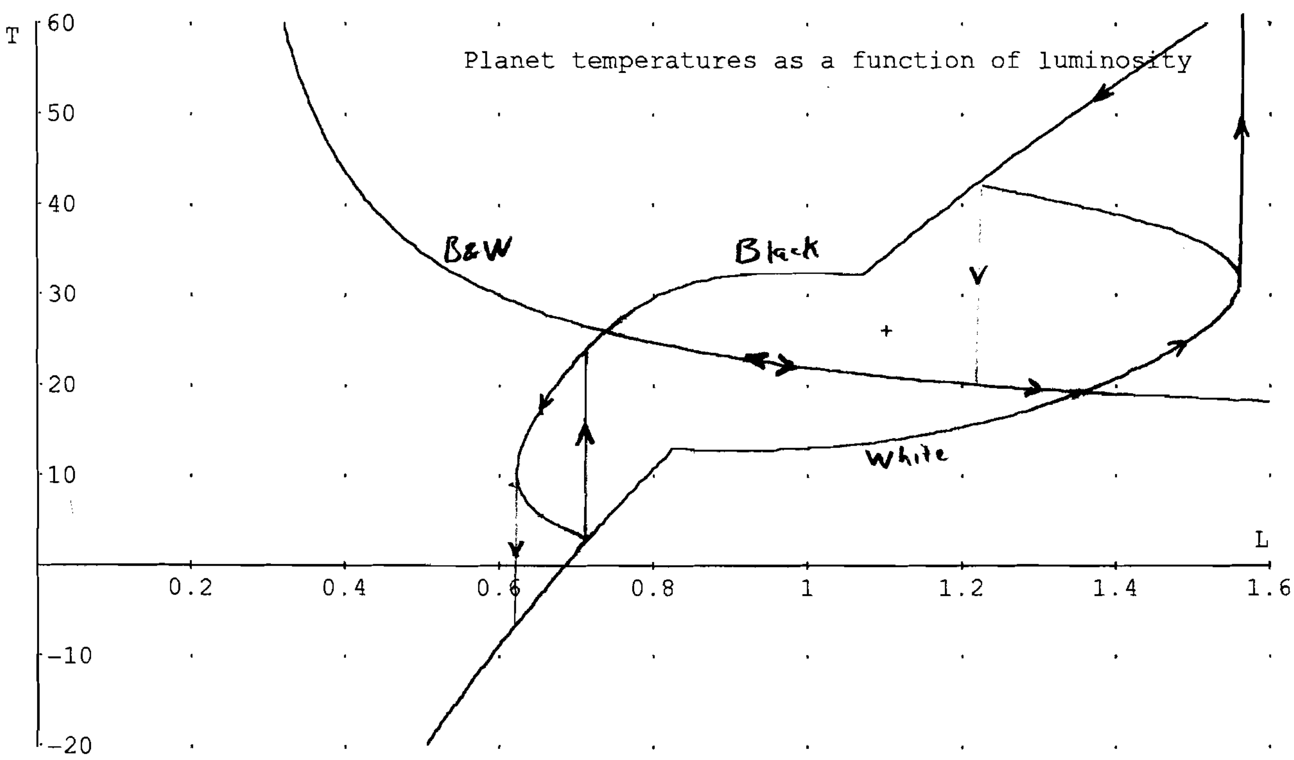
1.7
1.8

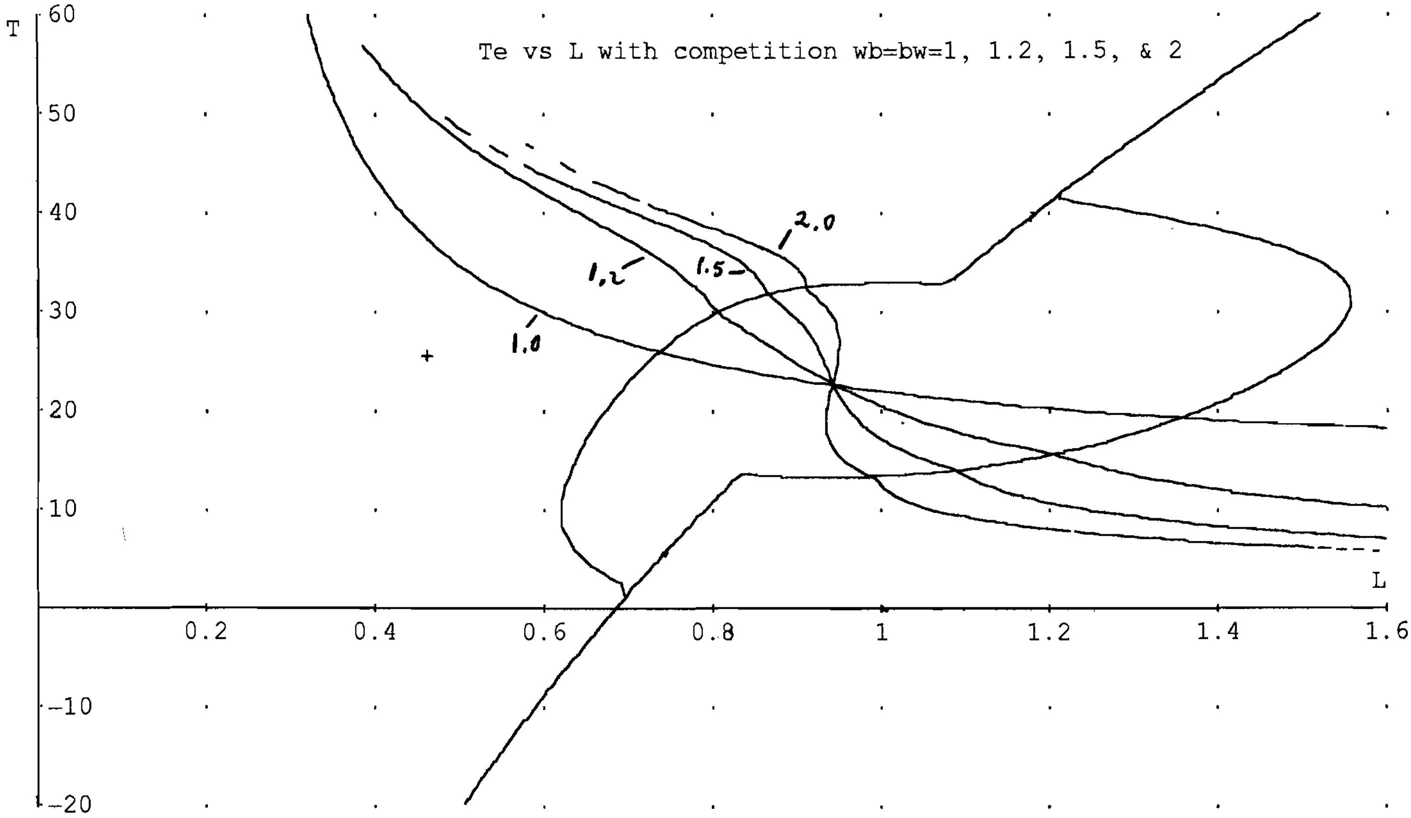


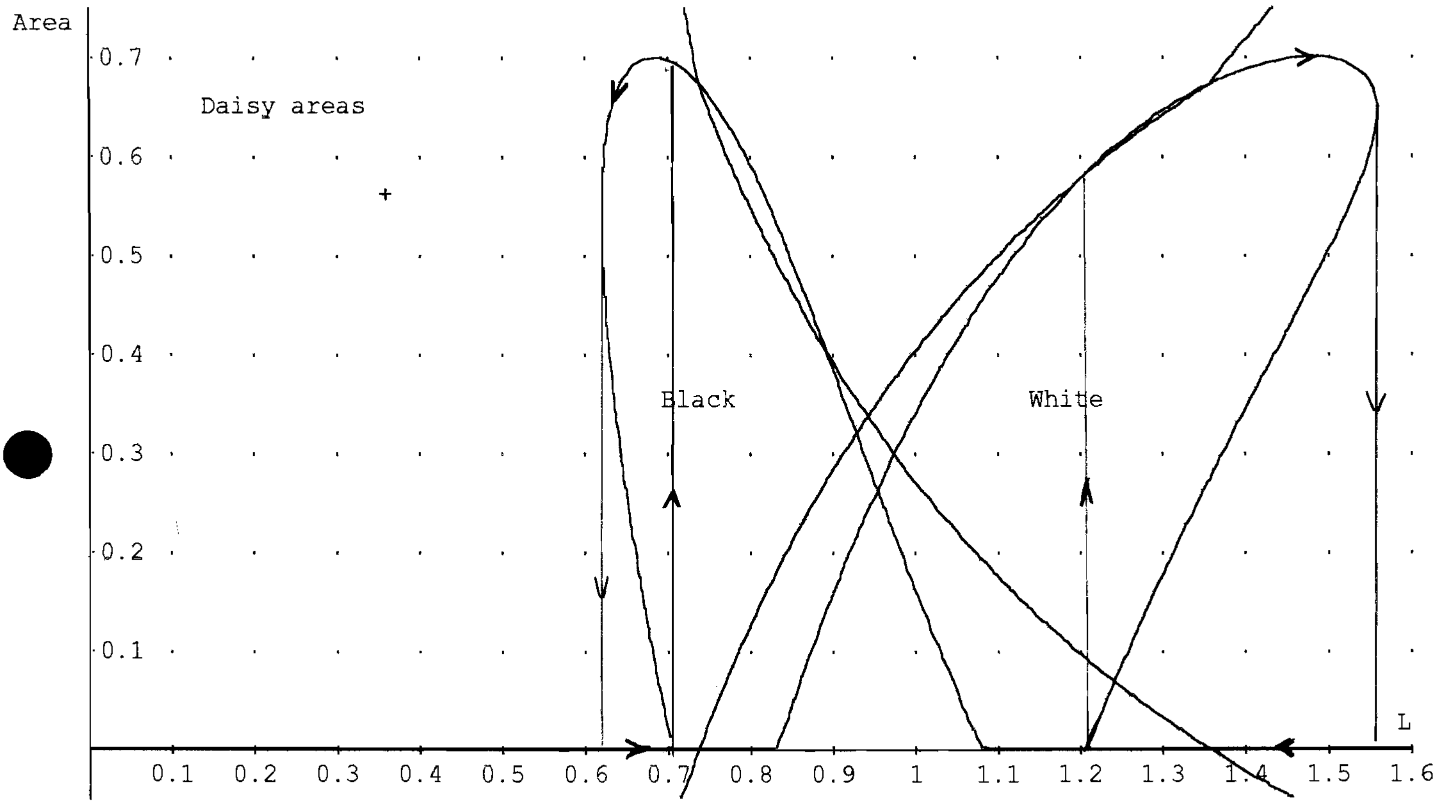
T/10A
50
40
30
20
10
-10
-20
bb=1 bw=1 ww=1 wb=1
γb=0.5 γw=0.1
Temperatures:
Growth rates:
Daisy areas:
L
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8


T/10A
50
40
30
20
10
-10
-20
bb=1 bw=1 ww=1 wb=1
γb=0.5 γw=0.3
Temperatures:
Growth rates:
Daisy areas:
L
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8

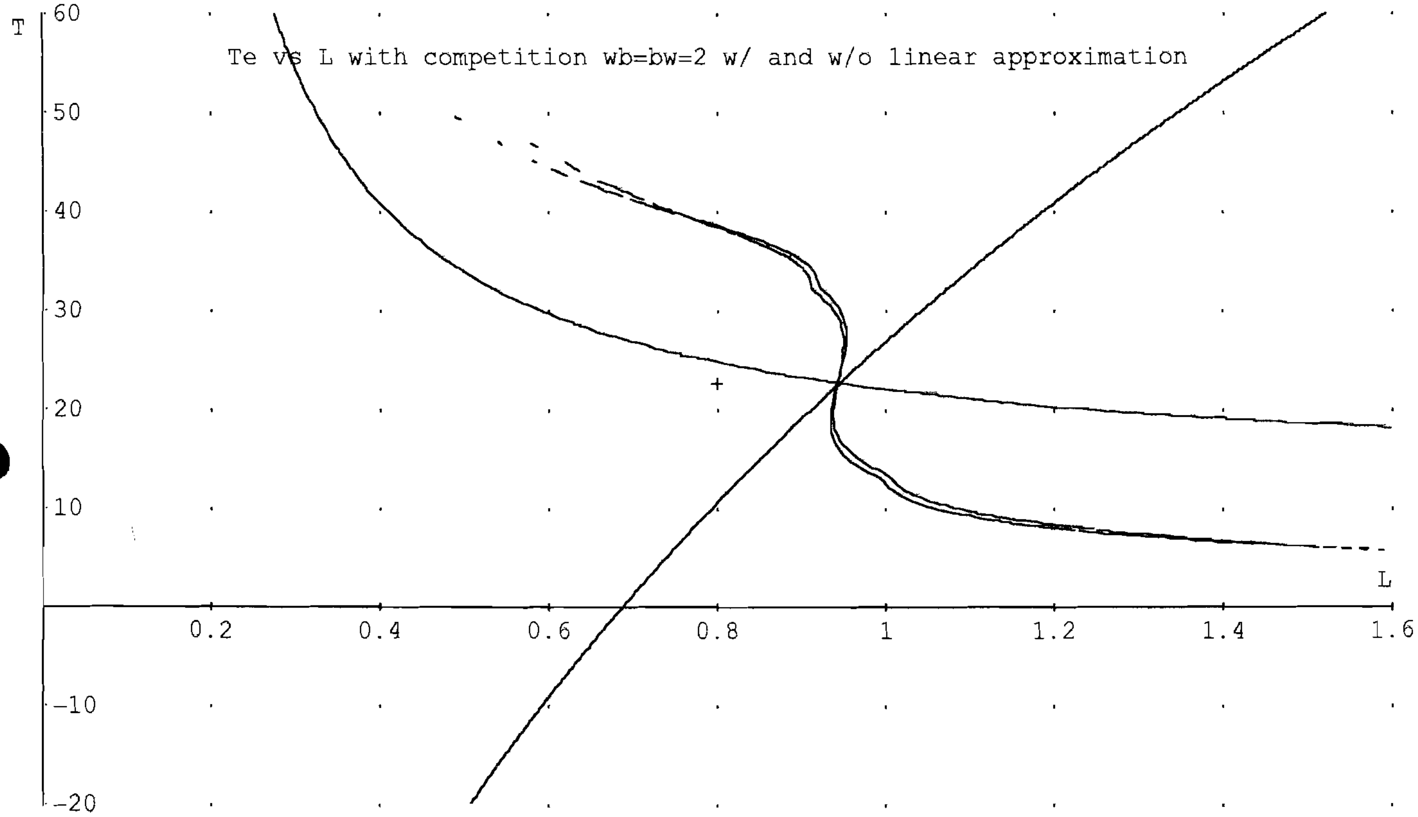

T/10A
50
40
30
20
10
-10
-20
bb=1 bw=1 ww=1 wb=1
γb=0.5 γw=0.5
Temperatures:
Growth rates:
Daisy areas:
L
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8

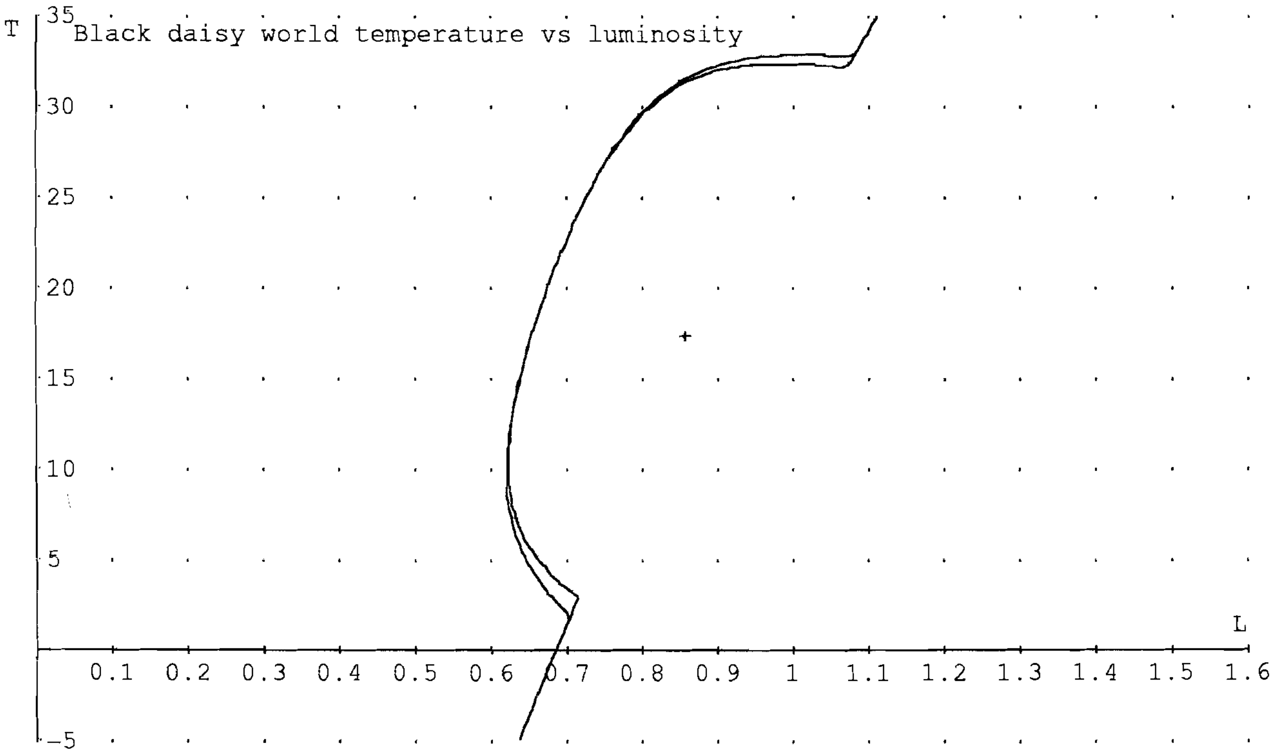

Planet temperatures as a function of luminosity
T
60
50
40
30
20
10
-10
-20
L
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
B&W
Black
White


T
60
50
40
30
20
10
-10
-20
Te vs L with competition wb=bw=1, 1.2, 1.5, & 2
2.0
1.5
1.2
1.0
L
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6

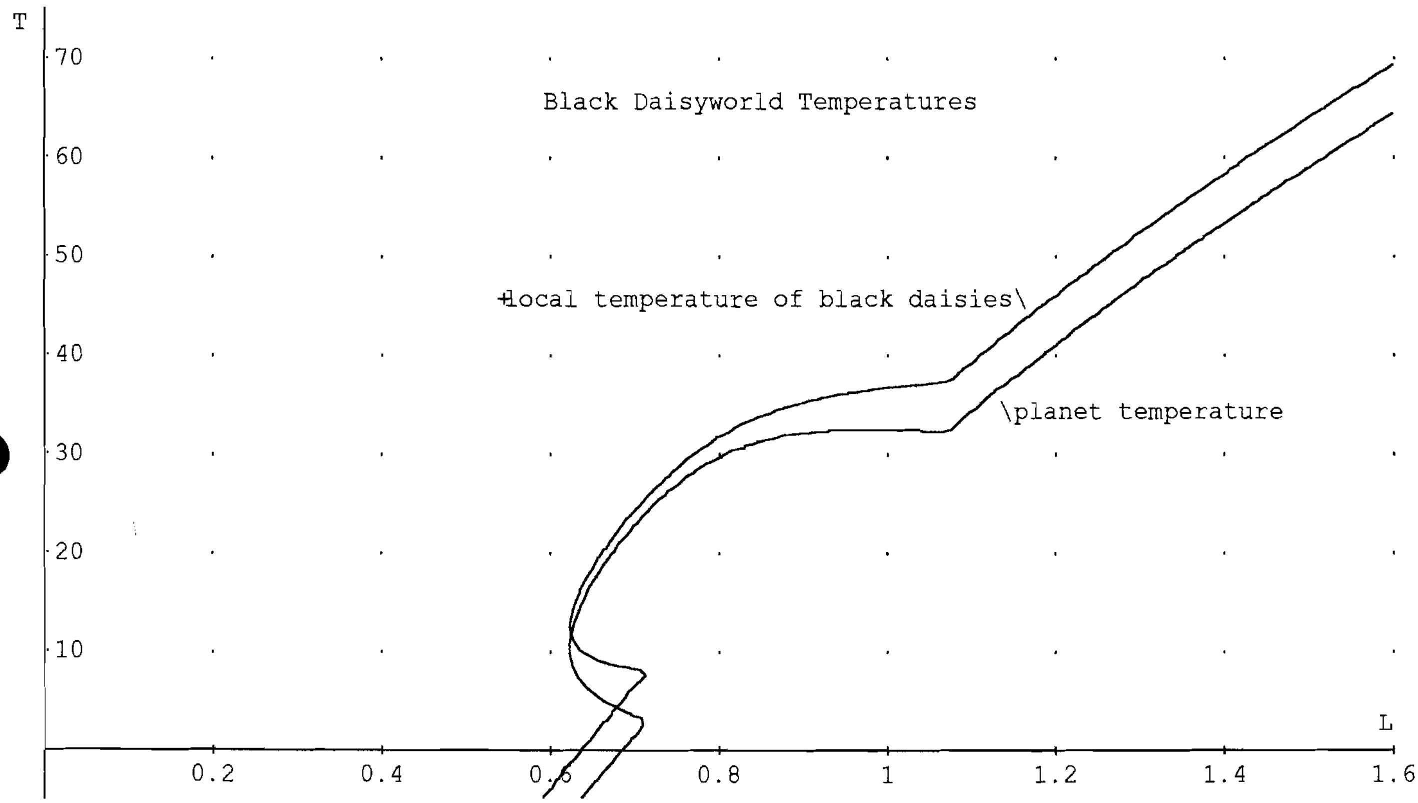


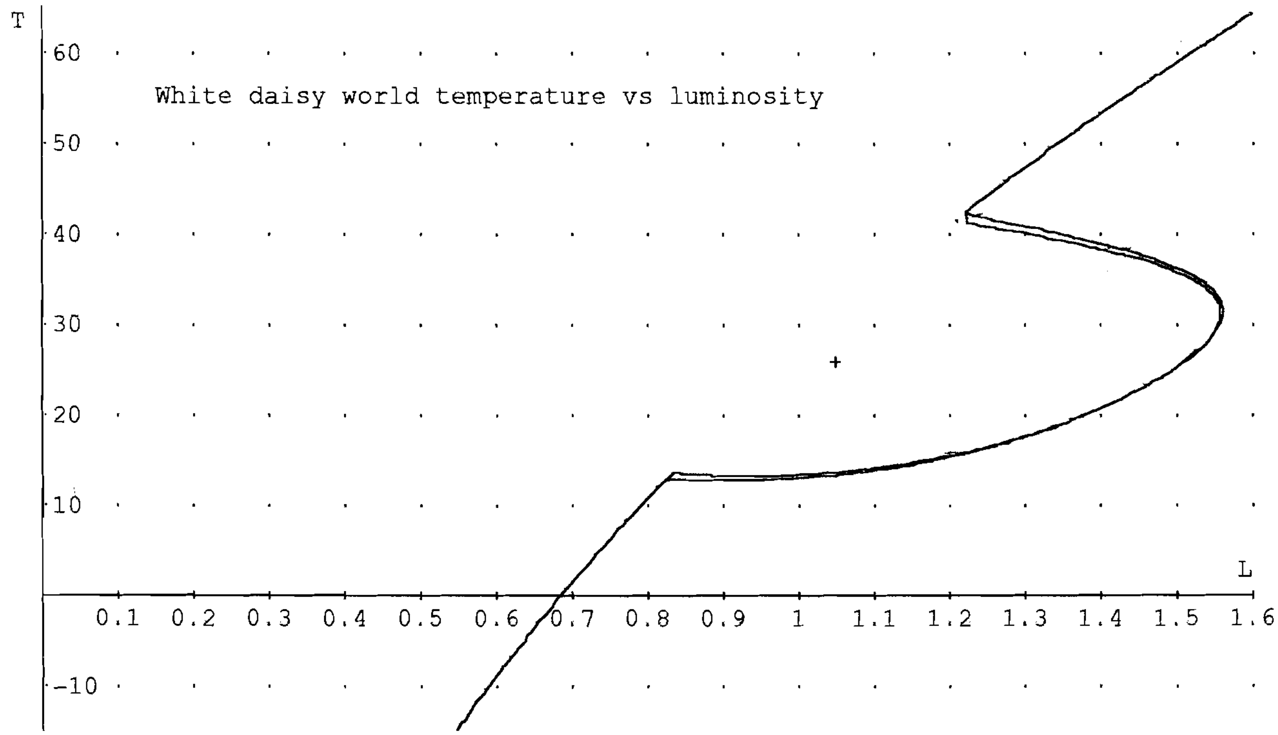
Indicates increasing ~~temperature~~ luminosity world.

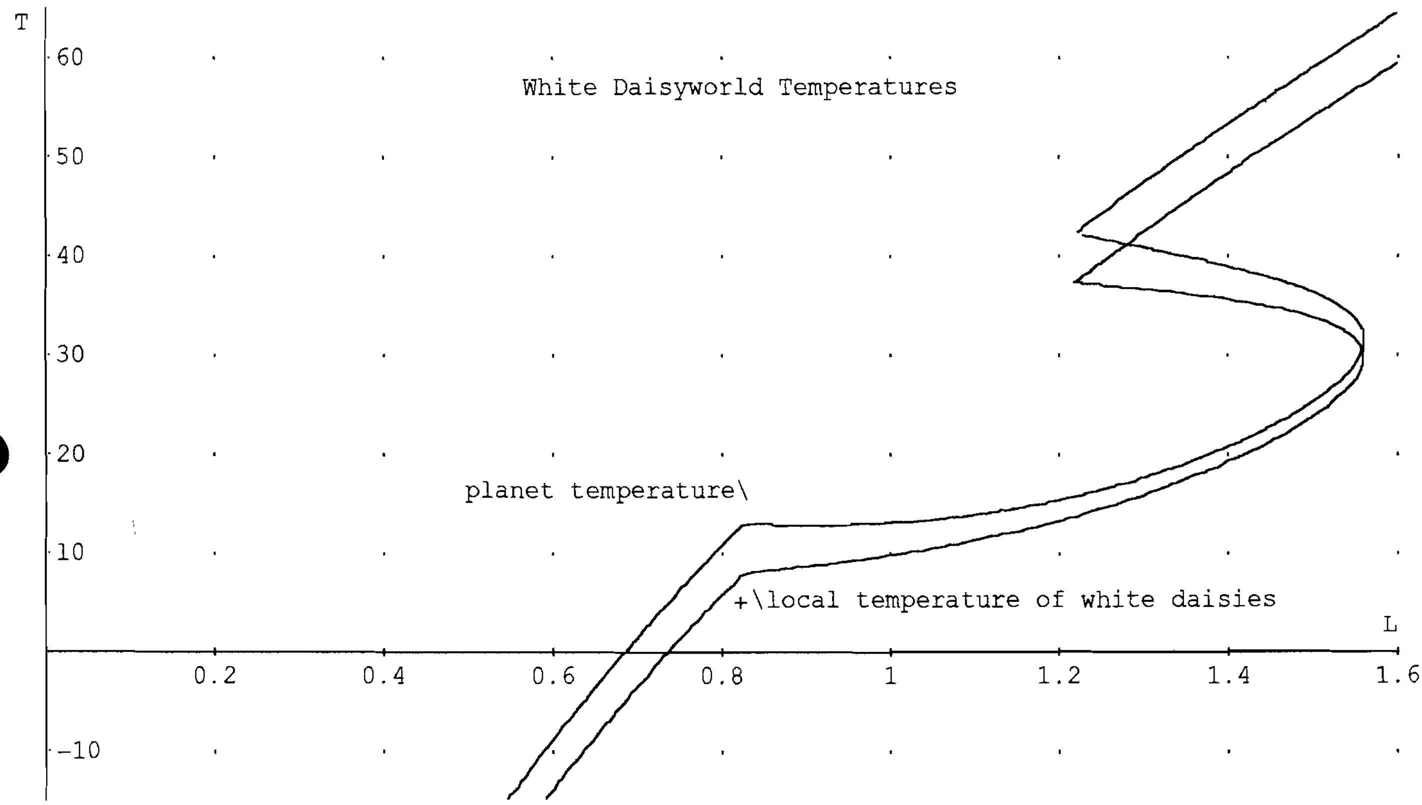

T
60
50
40
30
20
10
-10
-20
Te vs L with competition wb=bw=2 w/ and w/o linear approximation
L
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6

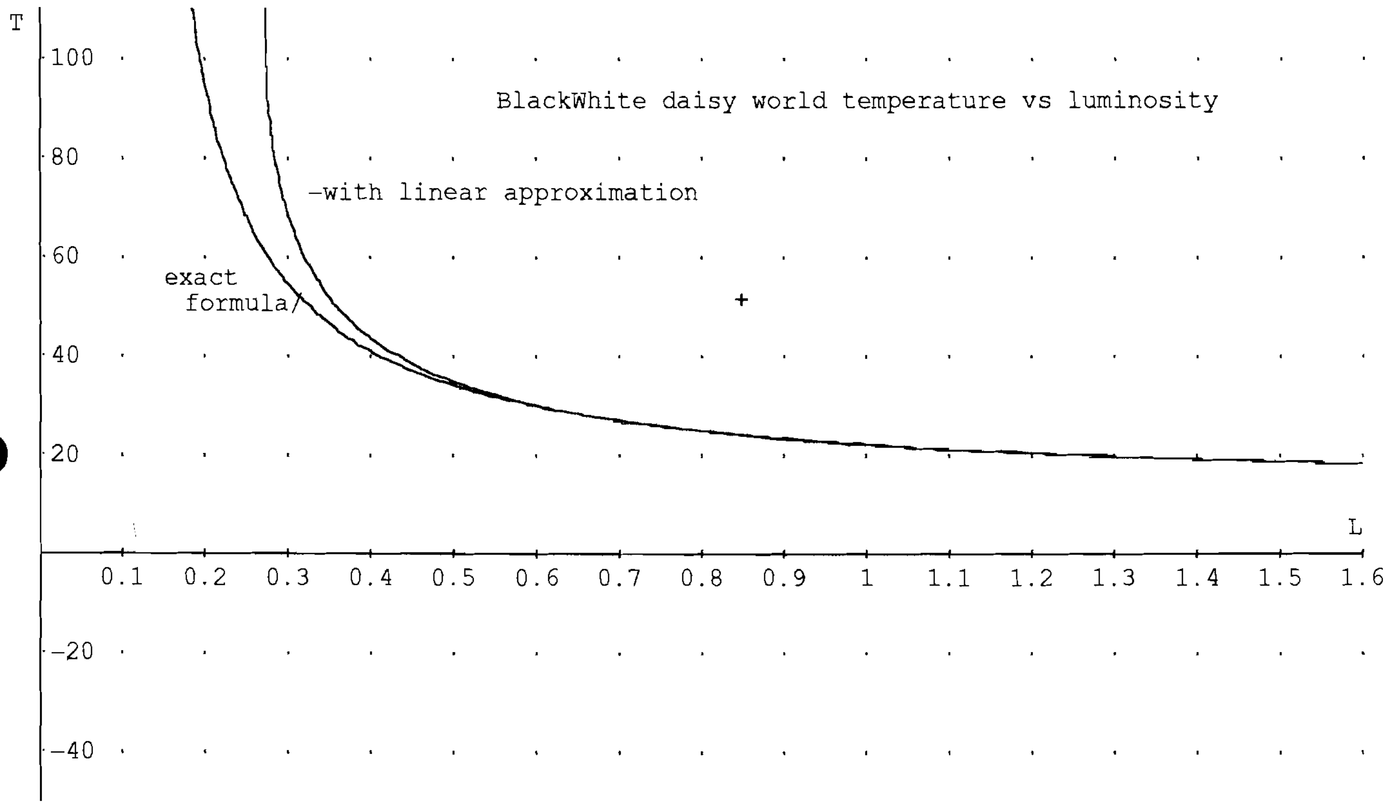


Black daisy world temperature vs luminosity
T
35
30
25
20
15
10
5
-5
L
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
1.1
1.2
1.3
1.4
1.5
1.6


T
70
60
50
40
30
20
10
Black Daisyworld Temperatures
+local temperature of black daisies\
\planet temperature
L
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6


T
60
50
40
30
20
10
-10
White daisy world temperature vs luminosity
L
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1 1.1 1.2 1.3 1.4 1.5 1.6


T
60
50
40
30
20
10
-10
White Daisyworld Temperatures
planet temperature\
+\local temperature of white daisies
L
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6


T
100
80
60
40
20
-20
-40
BlackWhite daisy world temperature vs luminosity
-with linear approximation
exact
formula
L
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
1.1
1.2
1.3
1.4
1.5
1.6

"Physical constants:"

$\sigma := 5.67 \cdot 10^{-\cancel{5}\,8}$ Stefan's constant

$S := 9.17 \cdot 10^{\cancel{5}\,2}$ Flux constant

"Daisyworld constants:"

$[Ag := 0.5,\ Ab := 0.25,\ Aw := 0.75,\ An := 0.5]$ Daisy albedos

$P := 1$ Fraction of planet's area that is fertile ground

$\gamma := 0.3$ Death rate

$qprime := 20$ q' – Solar energy redistribution constant

"Daisyworld equations:"

$q := 4 \cdot (273 + 22.5)^3 \cdot qprime$ Solar energy redistribution constant

$\alpha f := P - \alpha b - \alpha w - \alpha n$ Fraction of planet's area covered by fertile bare ground

$\alpha g := 1 - \alpha b - \alpha w - \alpha n$ Fraction of planet's area covered by bare ground

"Growth rate of daisies in term of local temperature"

$$\beta b := MAX\left(0,\ 1 - \left(\frac{22.5 - Tb}{17.5}\right)^2\right)$$

$$\beta w := MAX\left(0,\ 1 - \left(\frac{22.5 - Tw}{17.5}\right)^2\right)$$

$$\beta n := MAX\left(0,\ 1 - \left(\frac{22.5 - Tn}{17.5}\right)^2\right)$$

Effective temperature of a bare ground world in terms of luminosity

$$Teg := \left(\frac{S \cdot L \cdot (1 - Ag)}{\sigma}\right)^{1/4} - 273$$

"Linear approximation of local temperature in terms of albedo and planet temperature"

$Tb := qprime \cdot (A - Ab) + Te$

$Tw := qprime \cdot (A - Aw) + Te$

$Tn := qprime \cdot (A - An) + Te$

"Local temperature in terms of albedo and planet temperature"

$$Tb := \left(q \cdot (A - Ab) + (Te + 273)^4\right)^{1/4} - 273$$

$$Tw := \left(q \cdot (A - Aw) + (Te + 273)^4\right)^{1/4} - 273$$

$$Tn := \left(q \cdot (A - An) + (Te + 273)^4\right)^{1/4} - 273$$

"Daisy growth equations with competition:"

$$\frac{d}{dt}\alpha b = \alpha b\cdot((P - bb\cdot\alpha b - wb\cdot\alpha w)\cdot\beta b - \gamma)$$

$$\frac{d}{dt}\alpha w = \alpha w\cdot((P - bw\cdot\alpha b - ww\cdot\alpha w)\cdot\beta w - \gamma)$$

$$bb\cdot\alpha b + wb\cdot\alpha w = P - \frac{\gamma}{\beta b}$$

$$bw\cdot\alpha b + ww\cdot\alpha w = P - \frac{\gamma}{\beta w}$$

$$(bb\cdot ww - bw\cdot wb)\cdot\alpha b = ww\cdot\left(P - \frac{\gamma}{\beta b}\right) - wb\cdot\left(P - \frac{\gamma}{\beta w}\right)$$

$$(bw\cdot wb - bb\cdot ww)\cdot\alpha w = bw\cdot\left(P - \frac{\gamma}{\beta b}\right) - bb\cdot\left(P - \frac{\gamma}{\beta w}\right)$$

$$\alpha b := \mathrm{MAX}\left(0, \frac{ww\cdot\left(P - \frac{\gamma}{\beta b}\right) - wb\cdot\left(P - \frac{\gamma}{\beta w}\right)}{bb\cdot ww - bw\cdot wb}\right)$$

$$\alpha w := \mathrm{MAX}\left(0, \frac{bw\cdot\left(P - \frac{\gamma}{\beta b}\right) - bb\cdot\left(P - \frac{\gamma}{\beta w}\right)}{bw\cdot wb - bb\cdot ww}\right)$$

To plot temperature vs luminosity of a planet with competing black and white daisies

$$\left[\begin{array}{c} Te := \\ \alpha b := \dfrac{ww\cdot\left(P - \frac{\gamma}{\beta b}\right) - wb\cdot\left(P - \frac{\gamma}{\beta w}\right)}{bb\cdot ww - bw\cdot wb} \\ \alpha w := \dfrac{bw\cdot\left(P - \frac{\gamma}{\beta b}\right) - bb\cdot\left(P - \frac{\gamma}{\beta w}\right)}{bw\cdot wb - bb\cdot ww} \\ \alpha n := 0 \\ Tb := qprime\cdot(A - Ab) + Te \\ Tw := qprime\cdot(A - Aw) + Te \\ A := 1 - \dfrac{\sigma}{S\cdot L}\cdot(Te + 273)^4 \\ A = Ag + \alpha b\cdot(Ab - Ag) + \alpha w\cdot(Aw - Ag) \end{array}\right]$$

"At steady state with living black and white daisies:" User

$Tb + Tw = 45$ User

$Tb - Tw = qprime \cdot (Aw - Ab) = 10$ User

$Tb = 27.5$ User

$Tw = 17.5$ User

$\beta b = 1 - 0.003265 \cdot (22.5 - 27.5)^2 = 0.918375$ User

$\beta w = 1 - 0.003265 \cdot (22.5 - 17.5)^2 = 0.918375$ User

$\alpha b + \alpha w = MAX\left(0,\ P - \frac{\gamma}{\beta b}\right) = MAX\left(0,\ P - \frac{\gamma}{\beta w}\right) = 0.6733$ User

$\alpha g = 1 - (\alpha b + \alpha w) = 0.3267$ User

$A = \alpha g \cdot Ag + \alpha b \cdot Ab + \alpha w \cdot Aw = 0.3267 \cdot Ag + \alpha b \cdot Ab + \alpha w \cdot Aw$ User

"If αb=0, then αw=0.6733 and:" User

$A = 0.3267 \cdot Ag + 0.6733 \cdot Aw = 0.6683$ User

$Te = Tb - qprime \cdot (A - Ab) = 19.1335$ User

$L = \frac{\sigma \cdot (Te + 273)^4}{S \cdot (1 - A)} = 1.358$ User

"If αw=0, then αb=0.6733 and:" User

$A = 0.3267 \cdot Ag + 0.6733 \cdot Ab = 0.3317$ User

$Te = Tb - qprime \cdot (A - Ab) = 25.8664$ User

$L = \frac{\sigma \cdot (Te + 273)^4}{S \cdot (1 - A)} = 0.73816$ User

"Absolute minimum and maximum temperatures at which daisies can survive:"

$P \cdot \beta b - \gamma = 0$ At transition from black daisies to none

$P \cdot \beta w - \gamma = 0$ At transition from black daisies to none

$Tbcb = 22.5 - 17.5 \cdot \sqrt{\left(1 - \frac{\gamma}{P}\right)} = 7.85844$ Minimum black daisy local temperature

Minimum black daisy planet temperature

$Tecb = 22.5 - 17.5 \cdot \sqrt{\left(1 - \frac{\gamma}{P}\right)} - qprime \cdot (Ag - Ab) = 2.85844$

Minimum black daisy planet temperature

$Tecb = \left(q \cdot (Ab - Ag) + \left(22.5 - 17.5 \cdot \sqrt{\left(1 - \frac{\gamma}{P}\right)} + 273\right)^4\right)^{1/4} - 273 = 1.84459$

$$Twwh = 22.5 + 17.5\cdot\sqrt{\left(1 - \frac{Y}{P}\right)} = 37.1415$$

Maximum white daisy local temperature

Maximum white daisy planet temperature

$$Tewh = 22.5 + 17.5\cdot\sqrt{\left(1 - \frac{Y}{P}\right)} - qprime\cdot(Ag - Aw) = 42.1415$$

Maximum white daisy planet temperature

$$Tewh = \left(q\cdot(Aw - Ag) + \left(22.5 + 17.5\cdot\sqrt{\left(1 - \frac{Y}{P}\right)} + 273\right)^{4}\right)^{1/4} - 273 = 41.3786$$

To plot black daisy area vs luminosity of a planet with only black daisies

$$\alpha b :=$$

$$\alpha w := 0$$

$$\alpha n := 0$$

$$Te := \left(\frac{S \cdot L \cdot (1 - A)}{\sigma} \right)^{1/4} - 273$$

$$A := \alpha g \cdot Ag + \alpha b \cdot Ab + \alpha w \cdot Aw + \alpha n \cdot An$$

$$\alpha b = MAX\left(0,\ P - \frac{Y}{\beta b}\right)$$

To plot white daisy area vs luminosity of a planet with only white daisies

$$\alpha b := 0$$

$$\alpha w :=$$

$$\alpha n := 0$$

$$Te := \left(\frac{S \cdot L \cdot (1 - A)}{\sigma} \right)^{1/4} - 273$$

$$A := \alpha g \cdot Ag + \alpha b \cdot Ab + \alpha w \cdot Aw + \alpha n \cdot An$$

$$\alpha w = MAX\left(0,\ P - \frac{Y}{\beta w}\right)$$

To plot black daisy area vs luminosity of a planet with black and white daisies

$$\alpha b :=$$

$$\alpha w := MAX\left(0,\ P - \frac{Y}{\beta b} - \alpha b\right)$$

$$\alpha n := 0$$

$$Te := \left(\frac{S \cdot L \cdot (1 - A)}{\sigma} \right)^{1/4} - 273$$

$$A := \alpha g \cdot Ag + \alpha b \cdot Ab + \alpha w \cdot Aw + \alpha n \cdot An$$

$$Tb := 27.5$$

$$Tb = qprime \cdot (A - Ab) + Te$$

To plot white daisy area vs luminosity of a planet with black and white daisies

$$\left[\begin{array}{c} \alpha w \;:= \\ \alpha b \;:= \mathrm{MAX}\left(0,\; P - \dfrac{Y}{\beta w} - \alpha w\right) \\ \alpha n \;:= 0 \\ Te \;:= \left(\dfrac{S\cdot L\cdot(1 - A)}{\sigma}\right)^{1/4} - 273 \\ A \;:= \alpha g\cdot Ag + \alpha b\cdot Ab + \alpha w\cdot Aw + \alpha n\cdot An \\ TW \;:= 17.5 \\ Tw = qprime\cdot(A - Aw) + Te \end{array}\right]$$

To plot temperature vs luminosity of a planet with only neutral daisies

$$\left[\begin{array}{c} Te \;:= \\ A \;:= An \\ A = 1 - \dfrac{\sigma}{S\cdot L}\cdot(Te + 273)^{4} \end{array}\right]$$

To plot temperature vs luminosity of a planet with only black daisies

$$\left[\begin{array}{c} Te \;:= \\ \alpha b \;:= \dfrac{A - Ag}{Ab - Ag} \\ A \;:= 1 - \dfrac{\sigma}{S\cdot L}\cdot(Te + 273)^{4} \\ \alpha b = \mathrm{MAX}\left(0,\; P - \dfrac{Y}{\beta b}\right) \end{array}\right]$$

To plot temperature vs luminosity of a planet with only white daisies

$$\left[\begin{array}{c} Te \;:= \\ \alpha w \;:= \dfrac{A - Ag}{Aw - Ag} \\ A \;:= 1 - \dfrac{\sigma}{S\cdot L}\cdot(Te + 273)^{4} \\ \alpha w = \mathrm{MAX}\left(0,\; P - \dfrac{Y}{\beta w}\right) \end{array}\right]$$

To plot temperature vs luminosity of a planet with black and white daisies

$$\begin{bmatrix} Te := \\ A := 1 - \dfrac{\sigma}{S \cdot L} \cdot (Te + 273)^4 \\ 45 = Tb + Tw \end{bmatrix}$$

"Fig. 1b - A Daisyworld population of black and white daisies:"

$[\alpha b :=, \alpha w :=, \alpha n := 0]$ Areas in black and white daisy world

$\alpha f = P - \alpha b - \alpha w - \alpha n = 1 - \alpha b - \alpha w$ Area of fertile bare ground

$\alpha g = 1 - \alpha b - \alpha w - \alpha n = 1 - \alpha b - \alpha w$ Area of all bare ground

$A = \alpha g \cdot Ag + \alpha b \cdot Ab + \alpha w \cdot Aw + \alpha n \cdot An = 0.5 - 0.25 \cdot \alpha b + 0.25 \cdot \alpha w$ Albedo of planet

$Tb = qprime \cdot (A - Ab) + Te = 5 - 5 \cdot (\alpha b - \alpha w) + Te$ Local temperature of black daisies

$Tw = qprime \cdot (A - Aw) + Te = -5 - 5 \cdot (\alpha b - \alpha w) + Te$ Local temperature of white daisies

$Tb + Tw = 2 \cdot Te - 10 \cdot (\alpha b - \alpha w)$ Sum of local temperatures

Growth rate of black daisies

$$\beta b = MAX\left(0, 1 - 0.003265 \cdot (22.5 - Tb)^2\right) = MAX\left(0, 1 - 0.003265 \cdot (5 \cdot (\alpha b - \alpha w) - Te + 17.5)^2\right)$$

Growth rate of white daisies

$$\beta w = MAX\left(0, 1 - 0.003265 \cdot (22.5 - Tw)^2\right) = MAX\left(0, 1 - 0.003265 \cdot (5 \cdot (\alpha b - \alpha w) - Te + 27.5)^2\right)$$

Effective temperature of planet

$$Te = \left(\frac{S \cdot L \cdot (1 - A)}{\sigma}\right)^{1/4} - 273 = 252.163 \cdot (L \cdot (\alpha b - \alpha w + 2))^{1/4} - 273$$

Effective temperature in terms of luminosity and daisy areas

$$Te = 252.163 \cdot (L \cdot (\alpha b - \alpha w + 2))^{1/4} - 273$$

Difference of daisy areas in terms of luminosity and effective temperature

$$\alpha b - \alpha w = \frac{1}{L} \cdot \left(\frac{Te + 273}{252.163}\right)^4 - 2$$

$$\frac{d}{dt} \alpha b = 0 = \alpha b \cdot (\alpha f \cdot \beta b - \gamma)$$ Steady state black daisy growth equation

$$\frac{d}{dt} \alpha w = 0 = \alpha w \cdot (af \cdot \beta w - \gamma)$$ Steady state white daisy growth equation

$\alpha b = 0$ Strictly white daisy world solution

$\alpha w = 0$ Strictly black daisy world solution

$$\alpha b + \alpha w = MAX\left(0,\ 1 - \frac{\gamma}{\beta b}\right)$$

$$\alpha b + \alpha w = MAX\left(0,\ 1 - \frac{\gamma}{\beta w}\right)$$

$$\alpha b + \alpha w = MAX\left(0,\ 1 - \frac{0.3}{MAX\left(0,\ 1 - 0.003265\cdot(5\cdot(\alpha b - \alpha w) - Te + 17.5)^{2}\right)}\right)$$

$$\alpha b + \alpha w = MAX\left(0,\ 1 - \frac{0.3}{MAX\left(0,\ 1 - 0.003265\cdot(5\cdot(\alpha b - \alpha w) - Te + 27.5)^{2}\right)}\right)$$

$$\alpha b + \alpha w = MAX\left(0,\ 1 - \frac{0.3}{MAX\left(0,\ 1 - 0.003265\cdot\left(5\cdot(\alpha b - \alpha w) - 252.163\cdot(L\cdot(\alpha b - \alpha w + 2))^{1/4} + 290.5\right)^{2}\right)}\right)$$

$$\alpha b + \alpha w = MAX\left(0,\ 1 - \frac{0.3}{MAX\left(0,\ 1 - 0.003265\cdot\left(5\cdot(\alpha b - \alpha w) - 252.163\cdot(L\cdot(\alpha b - \alpha w + 2))^{1/4} + 300.5\right)^{2}\right)}\right)$$

$$\beta b = \beta w$$

$$Tb - 22.5 = 22.5 - Tw$$

$$Tb + Tw = 45 = 2\cdot Te - 10\cdot(\alpha b - \alpha w)$$

$$45 = 2\cdot Te - 10\cdot\left(\frac{1}{L}\cdot\left(\frac{Te + 273}{252.163}\right)^{4} - 2\right)$$

$$25 = 2\cdot Te - \frac{10}{L}\cdot\left(\frac{Te + 273}{252.163}\right)^{4}$$

"Derivation NOT using the linear approximation to the heat balance equation:"

$$q := 2.06425 \cdot 10^{9}$$

$$Tb = \left(q \cdot (A - Ab) + (Te + 273)^{4}\right)^{1/4} - 273$$

$$Tw = \left(q \cdot (A - Aw) + (Te + 273)^{4}\right)^{1/4} - 273$$

$$Tb + Tw = 45 = \left(q \cdot (A - Ab) + (Te + 273)^{4}\right)^{1/4} + \left(q \cdot (A - Aw) + (Te + 273)^{4}\right)^{1/4} - 546$$

$$A - Ab = 0.25 - 0.25 \cdot (\alpha b - \alpha w) = 0.25 - 0.25 \cdot \left(\frac{1}{L} \cdot \left(\frac{Te + 273}{252.163}\right)^{4} - 2\right)$$

$$A - Aw = -0.25 - 0.25 \cdot (\alpha b - \alpha w) = -0.25 - 0.25 \cdot \left(\frac{1}{L} \cdot \left(\frac{Te + 273}{252.163}\right)^{4} - 2\right)$$

$$45 = \left(q \cdot \left(0.25 - 0.25 \cdot \left(\frac{1}{L} \cdot \left(\frac{Te + 273}{252.163}\right)^{4} - 2\right)\right) + (Te + 273)^{4}\right)^{1/4} + \left(q \cdot \left(-0.25 - 0.25 \cdot \left(\frac{1}{L} \cdot \left(\frac{Te + 273}{252.163}\right)^{4} - 2\right)\right) + (Te + 273)^{4}\right)^{1/4} - 546$$